

MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2021

SET 7

Deadline: Wednesday 17 March, 2021, 23:59.

Exercise 1. *Lewis-Zax, 14.3. a,b,c,e.*

Exercise 2. *Let $X := \{2, 3, 4, 16\}$ be ordered by divisibility. Find the maximal and minimal elements of X .*

Exercise 3. *Consider the set $U := \{a, b, c, d\}$ with the relation*

$$P := \{(a, a), (b, b), (c, c), (d, d), (a, d), (c, a), (c, b), (c, d)\}$$

defined on it. Show that P is a poset and draw its corresponding Hasse diagram.

Exercise 4. *Consider the function $f(x) = 3x - 2$ with the integers as domain and codomain. Is it injective, surjective, or bijective?*

Exercise 5. *Let $f : \mathbb{Z} \rightarrow \mathbb{N}$ be defined by*

$$f(x) = \begin{cases} 2x - 1 & \text{if } x > 0 \\ -2x & \text{if } x \leq 0 \end{cases}$$

Prove that f is one-to-one and onto, and determine f^{-1} .

Exercise 6. *i) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Show that if $g \circ f : A \rightarrow C$ is surjective, then g is surjective.*

ii) Let $g : A \rightarrow B$ and $f : B \rightarrow C$ be functions. Show that if g and f are both injective, then $f \circ g : A \rightarrow C$ is injective.

Exercise 7. *Show that a function $f : A \rightarrow B$ is surjective if and only if given any functions $g, h : B \rightarrow C$, whenever $g \circ f = h \circ f$, then $g = h$.*

Exercise 8. *Let $f : A \rightarrow B$ be a function and $B_1, B_2 \subseteq B$. Prove that $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$ and $f^{-1}(\overline{B_1}) = \overline{f^{-1}(B_1)}$.*