

MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2021

SET 6

Deadline: Wednesday 18th March, 2021, 23:59.

Exercise 1. For each of the following determine if the relation is a partial order.

- a. R is the relation on \mathbb{Z} where xRy if $x - y$ is even.
 - b. R is the relation on $\mathbb{Z} \times \mathbb{Z}$ where $(a, b)R(c, d)$ if $a \leq c$.
- (Note: $R \subset (\mathbb{Z} \times \mathbb{Z}) \times (\mathbb{Z} \times \mathbb{Z})$)

Exercise 2. Let $A = \{1, 2, 3, 6, 9, 18\}$ and define R on A by xRy if $x|y$. Draw the Hasse diagram for the poset (A, R) .

Exercise 3. For $X = \{0, 1\}$ let $A = X \times X$. Define the relation R on A by $(a, b)R(c, d)$ if i) $a < c$; or ii) $a = c$ and $b \leq d$.

- a. Prove that R is a partial order for A .
- b. Determine all minimal and maximal elements of this partial order.
- c. Is this partial order a total order?

Exercise 4. Determine whether or not each of the following relations is a function. If the relation is a function, find its range.

- a. $\{(x, y) | x, y \in \mathbb{Z}, y = x^2 + 7\}$ a relation from \mathbb{Z} to \mathbb{Z} .
- b. $\{(x, y) | x, y \in \mathbb{R}, y^2 = x\}$ a relation from \mathbb{R} to \mathbb{R} .
- c. $\{(x, y) | x, y \in \mathbb{R}, y = 3x + 1\}$ a relation from \mathbb{R} to \mathbb{R} .
- d. $\{(x, y) | x, y \in \mathbb{Q}, x^2 + y^2 = 1\}$ a relation from \mathbb{Q} to \mathbb{Q} .

Exercise 5. For each of the following functions $f : \mathbb{Z} \rightarrow \mathbb{Z}$, determine whether the function is one-to-one and whether it is onto. Determine the range of $f(\mathbb{Z})$.

- a. $f(x) = 2x - 3$
- b. $f(x) = x^2$
- c. $f(x) = x^3 + x$

Exercise 6. For each of the following functions $g : \mathbb{R} \rightarrow \mathbb{R}$, determine whether the function is one-to-one and whether it is onto. Determine the range of $g(\mathbb{R})$.

- a. $f(x) = 2x - 3$
- b. $f(x) = x^2$
- c. $f(x) = x^3 + x$