

MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2021

SET 3

Deadline: Wednesday 10 February, 2021, 13:00.

Exercise 1. Use the laws of logic to simplify (give all steps):

$$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q).$$

Exercise 2. Use the laws of logic to simplify (give all steps):

$$((p \wedge q) \vee (p \wedge \neg r) \vee \neg(\neg p \vee q)) \vee ((r \vee s \vee \neg r) \wedge \neg q).$$

Exercise 3. Lewis-Zax, i) 5.1, ii) 5.4.

Exercise 4. Lewis-Zax, 5.7.

Exercise 5. Lewis-Zax, 5.9.

Exercise 6. i) If A and B are sets, we define their symmetric difference by

$$A\Delta B = (A \setminus B) \cup (B \setminus A).$$

Give a proof by elements for the equality

$$A\Delta B = (A \cup B) \setminus (A \cap B)$$

i.e. show that $x \in A\Delta B \Leftrightarrow x \in (A \cup B) \setminus (A \cap B)$ for any x .

ii) Suppose that $U = \mathbb{N} = \{1, 2, 3, 4, 5, 6, \dots\}$. Let $A = \{1, 2, 3, 4, 5, 6, 7, 9, 10\}$ and $B := \{1, 3, 5, 8, 9, 10\}$. Compute the symmetric difference $A\Delta B$.

Exercise 7.

Write down the power set, $\mathcal{P}(X)$ of the set $X := \{\{1, 2, 3\}, \{2, 3\}, \{e, f\}\} \cup \{\{e\}\}$ and compare with the power set, $\mathcal{P}(X \cap Y)$, where $Y := \{\{1, 2, 3, e, f\}\}$.

Exercise 8. A fundamental product of the sets A_1, A_2, \dots, A_n is defined to be a set of the form

$$A_1^{\epsilon_1} \cap A_2^{\epsilon_2} \cdots \cap A_n^{\epsilon_n}$$

where $A_i^{\epsilon_i}$ either the set A_i or its complement $\overline{A_i}$.

- (1) List all fundamental products of four sets A_1, A_2, A_3 .
- (2) Find the number of fundamental products of n sets A_1, A_2, \dots, A_n .

Exercise 9. Let A, B, C be sets. Use the laws of set algebra to show that

$$(A \setminus (B \setminus C)) \setminus ((A \setminus B) \setminus C) = A \cap C.$$

Exercise 10. Let A, B, C be sets. Use the laws of set algebra to show that

$$(A \cup B) \Delta (A \cup C) = (B \Delta C) \setminus A.$$