

MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2021

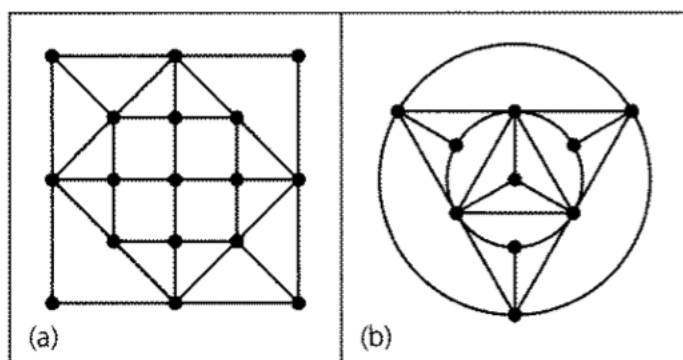
SET 11

Deadline: Wednesday 21st April, 2021, 23:59.

Exercise 1. Find all spanning trees of the complete graph K_4 .

Exercise 2. Lewis, Zax: Exercise 16.10a.

Exercise 3. Determine the number of vertices, edges, and regions for each of the planar graphs in the figure. Then show that your answers satisfy Euler's Theorem for connected planar graphs.



Exercise 4. Let $G = (V, E)$ be an undirected connected loop-free graph. Suppose further that G is planar and determines 53 regions. If, for some planar embedding of G , each region has at least five edges in its boundary, prove that $|V| \geq 82$.

Exercise 5. Determine whether or not the loop-free undirected graphs with the following adjacency matrices are isomorphic.

a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Exercise 6. Consider the words $u = abab$ and $v = bab$. Find:

- a) uv and $|uv|$
- b) vu and $|vu|$
- c) v^2 and $|v^2|$.

Date: April 15, 2021.

Exercise 7. Let $K = \{a, ab, a^2\}$ and $L = \{b^2, aba\}$ be languages over $A = \{a, b\}$. Find:

a) KL

b) LL

Exercise 8. Let $A = \{a, b, c\}$. Find L^* where:

a) $L = \{b^2\}$ b) $L = \{a, b\}$ c) $L = \{a, b, c^3\}$

Exercise 9. Let $A = \{a, b, c\}$ and let $w = abc$. State whether or not w belongs to $L(r)$ where:

a) $r = a^* \vee (b \vee c)^*$

b) $r = a^*(b \vee c)^*$