MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2020

SET 7

Exercise 1. *Let $\mathcal{C}$ be a collection of sets closed under the set operations of union, intersection, and complement. Show that $\mathcal{C}$ is a Boolean algebra.*

Exercise 2. *Let $(a_n)_{n\geq 0}$ be the sequence defined by $a_0 = 1$, $a_1 = 1$, $a_2 = 3$ and for $n > 0$, $a_{n+2} = 3a_{n+1} - 3a_n + a_{n-1}$. Use induction to show that for $n \geq 0$, $a_n = n^2 - n + 1$.*

Exercise 3. *Lewis–Zax (page 148/149): exercise 14.7.*

Exercise 4. *Consider the natural numbers $\mathbb{N}$ and define two relations R_1 and R_2:*

$$R_1 = \{(0,2),(0,5),(0,9),(1,9),(1,12),(1,15),(2,2)\} \subset \mathbb{N} \times \mathbb{N}$$

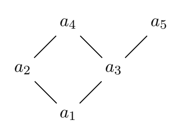
and $R_2 = \{(2,0),(2,6),(5,6),(9,8),(12,1),(12,7),(15,4)\} \subset \mathbb{N} \times \mathbb{N}$. Computer the relation $R_3 = R_1 \circ R_2$ and determine the inverse relations R_1^{-1}, R_2^{-1} and R_3^{-1}. Deduce a connection between the three inverse relations.

Exercise 5. *Consider the set $X := \{a_1, a_2, a_3, a_4, a_5\}$ and show that the relation*

$$R = \{(a_1,a_1),(a_2,a_2),(a_3,a_3),(a_4,a_4),(a_5,a_5),(a_1,a_2),(a_1,a_3),(a_1,a_4),(a_1,a_5),(a_2,a_3),(a_5,a_4),(a_5,a_3)\}$$

defines a partial order on X. Draw the corresponding Hasse diagram.

Exercise 6. *Given the set $A = \{a_1, a_2, a_3, a_4, a_5\}$ with an ordering described by the following diagram*



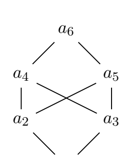
Write down all subsets of A in which the element a_3 is a maximal element.

Exercise 7. *Consider the set $X = \{1, 2, 4, 5, 10, 15, 20\}$ and the "divides" relation on it, xRy if and only if x divides y (that is, $y = zx$ for some $z \in \mathbb{Z}$), for $x, y \in X$. Draw the Hasse diagram.*

Exercise 8. *Let $X := \{2, 3, 4, 16\}$ be ordered by divisibility. Find the maximal and minimal elements of X.*

Exercise 9. *Consider the integers $\mathbb{Z}$ and define the relation R: for all $a, b \in \mathbb{Z}$, aRb if and only if $a + b$ is an even number. Prove or disprove (by a counterexample) that R is a partial ordering relation.*

Exercise 10. *Defines the following partially ordered set a lattice? Provide an argument.*



Date: February 21, 2020.