**MA0301**
**ELEMENTARY DISCRETE MATHEMATICS**
**NTNU, SPRING 2020**

## Set 4

**Exercise 1.** *Let $X$ and $Y$ be two sets. Recall the definition of power set of a set. Show that*

$$\mathcal{P}(X \cap Y) = \mathcal{P}(X) \cap \mathcal{P}(Y)$$

**Exercise 2.** *a) Let $P$ and $Q$ be two open statements with respect to the universe $U$. Recall the definition of truth set for an open statement. The truth sets for $P$ and $Q$ are denoted as $T_P$ respectively $T_Q$. Show the following: $T_P \cap T_Q = T_{P \wedge Q}$.*

*b) The falsity set of an open statement $P$ is denoted as $F_P$. It contains by definition all values in the universe $U$ such that $P \equiv F$. Let $P$ and $Q$ be two open statements with respect to the universe $U$. The falsity sets for $P$ and $Q$ are denoted as $F_P$ respectively $F_Q$. Show that: $F_P \cup F_Q = F_{P \wedge Q}$.*

**Exercise 3.** *Let the universe be the integers, $U = \mathbb{Z}$. Define $P(x) : x \text{ is even}$, with respect to the universe $U$. a) Paraphrase as a theorem the following open statement $P(xy) \Rightarrow (P(x) \vee P(y))$. b) Prove the theorem by checking the three possible cases.*

**Exercise 4.** *Let $B$ be a Boolean algebra and $x, y \in B$. Use Boolean algebra to show that: $x + \bar{x}y = x + y$.*

**Exercise 5.** *Let $B$ be a Boolean algebra and $x, y, z \in B$. Use Boolean algebra to show that: $(x + y)(x + z) = x + yz$*

**Exercise 6.** *Let $B$ be a Boolean algebra and $x, y, z \in B$. Use Boolean algebra to simplify the expression: $xy + x(y + z) + y(y + z)$*

**Exercise 7.** *Let $B$ be a Boolean algebra and $x, y, z, u \in B$. Use Boolean algebra to simplify the expression: $(\overline{\bar{x}\bar{y}\bar{z}})z + (\overline{\bar{x}\bar{y}\bar{z}}) + u$*

**Exercise 8.** *Let $B$ be a Boolean algebra. Prove for $x, y \in B$ that $x \cdot \bar{y} = 0$ if and only if $x \cdot y = x$.*

**Exercise 9.** *Let $B$ be a Boolean algebra. For $x, y, z \in B$ find the dual expressions of*

$$i)\ y \cdot x \cdot \bar{z} + x \cdot \bar{y} \cdot z$$

$$ii)\ x \cdot \bar{y} + x \cdot \bar{z} + y \cdot \bar{x}$$

$$iii)\ x \cdot y \cdot (0 + x + (z \cdot 1))$$

**Exercise 10.** *Let $B$ be a Boolean algebra. Prove for $x, y, z \in B$ that if $x \cdot y = x \cdot z$ and $\bar{x} \cdot y = \bar{x} \cdot z$, then $y = z$.*

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