MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2020

Set 2

Table 2.19

Rule of Inference	Related Logical Implication	Name of Rule
1) p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Rule of Detachment (Modus Ponens)
2) $p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Law of the Syllogism
3) $p \to q$ $\neg q$ $\therefore \neg p$	$[(p \to q) \wedge \neg q] \to \neg p$	Modus Tollens
4) p q $\therefore p \wedge q$		Rule of Conjunction
5) $p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \to q$	Rule of Disjunctive Syllogism
6) $\neg p \to F_0$ $\therefore p$	$(\neg p \to F_0) \to p$	Rule of Contradiction
7) $p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Rule of Conjunctive Simplification
8) p $\therefore p \vee q$	$p \to p \vee q$	Rule of Disjunctive Amplification
9) $p \wedge q$ $p \to (q \to r)$ $\therefore r$	$[(p \wedge q) \wedge [p \to (q \to r)]] \to r$	Rule of Conditional Proof
10) $p \to r$ $q \to r$ $\therefore (p \vee q) \to r$	$[(p \to r) \wedge (q \to r)] \to [(p \vee q) \to r]$	Rule for Proof by Cases
11) $p \to q$ $r \to s$ $p \vee r$ $\therefore q \vee s$	$[(p \to q) \wedge (r \to s) \wedge (p \vee r)] \to (q \vee s)$	Rule of the Constructive Dilemma
12) $p \to q$ $r \to s$ $\neg q \vee \neg s$ $\therefore \neg p \vee \neg r$	$[(p \to q) \wedge (r \to s) \wedge (\neg q \vee \neg s)] \to (\neg p \vee \neg r)$	Rule of the Destructive Dilemma

The table was taken from Grimaldi, *Discrete and Combinatorial Mathematics*, 5. edition, page 78. Recall that $\to$ equals $\Rightarrow$ in our notation.

Exercise 1. *Use truth tables to determine which of the following statements are tautologies and which are contradictions:*

$$a)\ q \vee (q \Rightarrow \neg q) \qquad b)\ \neg((\neg r \wedge r) \Rightarrow s) \qquad c)\ ((t \Rightarrow s) \Rightarrow t) \Rightarrow t$$

Exercise 2. *Check whether the following inference rules are sound:*

$$a)\ (p \Rightarrow q) \wedge (p \vee \neg r) \wedge (\neg r) \Rightarrow (\neg p) \qquad b)\ ((p \wedge q) \Rightarrow r) \wedge \neg(p \Rightarrow r) \Rightarrow (q \Rightarrow r)$$

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Exercise 3. *Negate the following statements:*

$$a)\ \neg r \wedge (r \vee \neg q) \qquad b)\ s \Rightarrow (\neg t \Leftrightarrow r)$$

Exercise 4. *Lewis–Zax (page 131): exercise 12.3. a) and b).*

Exercise 5. *Provide the reasons for each step (using inference rules) required to verify that the following argument is valid:*

$$\big(p \wedge (p \Rightarrow q) \wedge (s \vee r) \wedge (r \Rightarrow \neg q)\big) \Rightarrow (s \vee t)$$

Exercise 6. *Provide the reasons for each step (using inference rules) required to verify that the following argument is valid:*

$$\begin{array}{l} (\neg p \vee q) \Rightarrow r \\ r \Rightarrow (s \vee t) \\ \neg s \wedge \neg u \\ \neg u \Rightarrow \neg t \\ \hline \therefore p \end{array}$$

Exercise 7. *Show that the following argument is invalid by providing a set of truth values for the primitive statements p, q, r, s such that premises are true while the conclusion is false.*

$$\begin{array}{l} p \\ p \Rightarrow r \\ p \Rightarrow (q \vee \neg r) \\ \neg q \vee \neg s \\ \hline \therefore s \end{array}$$

Exercise 8. *Let the universe be the real numbers, $U = \mathbb{R}$. Negate and simplify:*

$$a)\ \forall x \forall y\ [(x > y) \Rightarrow (x - y > 0)] \qquad b)\ \forall x \forall y\ [(x < y) \Rightarrow \exists z\ (x < z < y)]$$

Exercise 9. *Prove that $\forall x\ p(x) \vee \forall x\ q(x)$ logically implies $\forall x\ (p(x) \vee q(x))$, where $p(x), q(x)$ are open statements with respect to the variable x from the universe U. Is the opposite statement true as well?*

Exercise 10. *Translate the following English phrases into statements (for real numbers in $\mathbb{R}$; use that $I(x)$ means that: x is an integer, i.e., $x \in \mathbb{Z}$.):*

(1) *Every square of an integer is greater than zero.*
(2) *Every integer is even or odd.*
(3) *There is a real number that, when multiplied by any real number, produces that number.*
(4) *For every integer there is a greater integer.*

Recall that an even integer writes $m = 2k$, whereas an odd integer writes $q = 2s + 1$, where k, are integers.