

MA0301
 ELEMENTARY DISCRETE MATHEMATICS
 NTNU, SPRING 2020

SET 2

You can deliver the exercises (before 10:30am on the indicated day) on the 3 floor of the central building, in an area called Matteland. The box number is 0613.

If you want feedback on each exercise, write "want comments"/"nsker retting", otherwise your TA will only write "godkjent"/"ikke godkjent".

Table 2.19

Rule of Inference	Related Logical Implication	Name of Rule
1) $\frac{p}{p \rightarrow q}$ $\therefore q$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Rule of Detachment (Modus Ponens)
2) $\frac{p \rightarrow q}{q \rightarrow r}$ $\therefore p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Law of the Syllogism
3) $\frac{p \rightarrow q}{\neg q}$ $\therefore \neg p$	$[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$	Modus Tollens
4) $\frac{p}{q}$ $\therefore p \wedge q$		Rule of Conjunction
5) $\frac{p \vee q}{\neg p}$ $\therefore q$	$[(p \vee q) \wedge \neg p] \rightarrow q$	Rule of Disjunctive Syllogism
6) $\frac{\neg p \rightarrow F_0}{\therefore p}$	$(\neg p \rightarrow F_0) \rightarrow p$	Rule of Contradiction
7) $\frac{p \wedge q}{\therefore p}$	$(p \wedge q) \rightarrow p$	Rule of Conjunctive Simplification
8) $\frac{p}{\therefore p \vee q}$	$p \rightarrow p \vee q$	Rule of Disjunctive Amplification
9) $\frac{p \wedge q}{p \rightarrow (q \rightarrow r)}$ $\therefore r$	$[(p \wedge q) \wedge [p \rightarrow (q \rightarrow r)]] \rightarrow r$	Rule of Conditional Proof
10) $\frac{p \rightarrow r}{q \rightarrow r}$ $\therefore (p \vee q) \rightarrow r$	$[(p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [(p \vee q) \rightarrow r]$	Rule for Proof by Cases
11) $\frac{p \rightarrow q}{r \rightarrow s}$ $\frac{p \vee r}{\therefore q \vee s}$	$[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (q \vee s)$	Rule of the Constructive Dilemma
12) $\frac{p \rightarrow q}{r \rightarrow s}$ $\frac{\neg q \vee \neg s}{\therefore \neg p \vee \neg r}$	$[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (\neg q \vee \neg s)] \rightarrow (\neg p \vee \neg r)$	Rule of the Destructive Dilemma

The table was taken from Grimaldi, *Discrete and Combinatorial Mathematics*, 5. edition, page 78. Recall that \rightarrow equals \Rightarrow in our notation.

Exercise 1. Use truth tables to determine which of the following statements are tautologies and which are contradictions:

$$a) q \vee (q \Rightarrow \neg q) \quad b) \neg((\neg r \wedge r) \Rightarrow s) \quad c) ((t \Rightarrow s) \Rightarrow t) \Rightarrow t$$

Exercise 2. Check whether the following inference rules are sound:

$$a) (p \Rightarrow q) \wedge (p \vee \neg r) \wedge (\neg r) \Rightarrow (\neg p) \quad b) ((p \wedge q) \Rightarrow r) \wedge \neg(p \Rightarrow r) \Rightarrow (q \Rightarrow r)$$

Exercise 3. Negate the following statements:

$$a) \neg r \wedge (r \vee \neg q) \quad b) s \Rightarrow (\neg t \Leftrightarrow r)$$

Exercise 4. Lewis–Zax (page 131): exercise 12.3. a) and b).

Exercise 5. Provide the reasons for each step (using inference rules) required to verify that the following argument is valid:

$$(p \wedge (p \Rightarrow q) \wedge (s \vee r) \wedge (r \Rightarrow \neg q)) \Rightarrow (s \vee t)$$

Exercise 6. Provide the reasons for each step (using inference rules) required to verify that the following argument is valid:

$$\begin{array}{l} (\neg p \vee q) \Rightarrow r \\ r \Rightarrow (s \vee t) \\ \neg s \wedge \neg u \\ \neg u \Rightarrow \neg t \\ \hline \therefore p \end{array}$$

Exercise 7. Show that the following argument is invalid by providing a set of truth values for the primitive statements p, q, r, s such that premises are true while the conclusion is false.

$$\begin{array}{l} p \\ p \Rightarrow r \\ p \Rightarrow (q \vee \neg r) \\ \neg q \vee \neg s \\ \hline \therefore s \end{array}$$

Exercise 8. Let the universe be the real numbers, $U = \mathbb{R}$. Negate and simplify:

$$a) \forall x \forall y [(x > y) \Rightarrow (x - y > 0)] \quad b) \forall x \forall y [(x < y) \Rightarrow \exists z (x < z < y)]$$

Exercise 9. Prove that $\forall x p(x) \vee \forall x q(x)$ logically implies $\forall x (p(x) \vee q(x))$, where $p(x), q(x)$ are open statements with respect to the variable x from the universe U . Is the opposite statement true as well?

Exercise 10. Translate the following English phrases into statements (for real numbers in \mathbb{R} ; use that $I(x)$ means that: x is an integer, i.e., $x \in \mathbb{Z}$):

- (1) Every square of an integer is greater than zero.
- (2) Every integer is even or odd.
- (3) There is a real number that, when multiplied by any real number, produces that number.
- (4) For every integer there is a greater integer.

Recall that an even integer writes $m = 2k$, whereas an odd integer writes $q = 2s + 1$, where k, s are integers.