MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2020

Set 1

Exercises are to be handed in and will be graded (godkjent / ikke godkjent). If you want feedback on each exercise, write "want comments"/"nsker retting", otherwise your TA will only write "godkjent"/"ikke godkjent". The exercise sets are mandatory; you need to pass eight (8) out of a total of twelve (12) sets in order to take the exam. Of those 8 solution sets, 4 must be with respect to the exercise sets 1-6 and 4 must be with respect to the exercise sets 7-12. Note that to get an exercise set approved you are required to have solved correctly at least 70% (providing detailed arguments/computations) and made serious and substantial attempts at solving at least 90% of a given exercise set.

You are asked to pay attention to the quality of presentation, in particular, the correctness of mathematical notation and formalism.

10 Problems

Exercise 1. *a) Write down the truth table for* $p \wedge (\neg p \wedge q)$.

b) Write down the truth tables for $p \wedge (q \vee r)$ *and* $(p \wedge q) \vee (p \wedge r)$. *What is your conclusion? What can you say about the two compound propositions* $p \vee (q \wedge r)$ *and* $(p \vee q) \wedge (p \vee r)$*?*

Exercise 2. *Use truth tables to:*

a) Verify that $\neg(p \wedge q)$ *and* $\neg p \vee \neg q$ *are logically equivalent.*

b) Verify that $p \vee (q \vee r)$ *and* $(p \vee q) \wedge r)$ *are not logically equivalent.*

Exercise 3. *1) Write down the truth table of the so-called EXCLUSIVE OR:* $p \oplus q$*, which is defined to be true if either* p *is true and* q *is false, or* p *is false and* q *is true, and it is false in all other cases.*

2) Verify that $p \oplus q$ *is logically equivalent to* $(p \wedge \neg q) \vee (\neg p \wedge q)$.

Exercise 4. *Write down the truth table for* $(p \Rightarrow q) \Rightarrow (p \wedge q)$.

Exercise 5. *Write down the truth table for* $(p \wedge q) \Rightarrow (p \vee q)$ *and draw a conclusion.*

Exercise 6. *Write down the truth table for* $(p \Leftrightarrow \neg q) \Leftrightarrow (q \Rightarrow p)$.

Exercise 7. *Use a truth table to show that* $\neg(p \Rightarrow q)$ *is logically equivalent to* $p \wedge \neg q$.

Date: January 10, 2020.

Exercise 8. *Use the laws of logic to show that* $(p \wedge q) \vee \neg p \equiv \neg p \vee q$.

Exercise 9. *Use the laws of logic to simplify* $\big(s \vee (p \wedge r \wedge s)\big) \wedge \big(p \vee (p \wedge q \wedge \neg r) \vee (p \wedge q)\big)$.

Exercise 10. *Definition: A compound statement is* **satisfiable** *if there is an assignment of truth values that makes it true. When no such assignment exists, i.e., when the compound statement is false for all assignments of truth values, then the compound proposition is* **unsatisfiable**. *In this sense, one can say that finding a particular assignment of truth values that makes a compound statement true amounts to giving a solution to this compound statement.*

a) Show that a compound proposition is unsatisfiable if and only if its negation is true for all assignments of truth values to the variables, i.e., if and only if its negation is a tautology.

b) Use truth tables to determine which of the following compound propositions is satisfiable, a tautology, or unsatisfiable: (i) $p \Rightarrow (p \vee q)$, *(ii)* $\neg(p \Rightarrow (p \vee t))$