MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2020

Set 12

Exercise 1. *Let B be a Boolean algebra. For $x, y, z \in B$ find the dual expressions of*

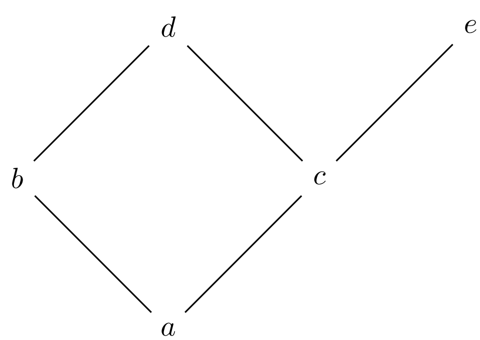
$$i)\ (x+\bar{y})\cdot\overline{(\bar{z}+y)} \qquad ii)\ (1+x)\cdot y + x\cdot\bar{y}\cdot z \qquad iii)\ (x\cdot y+1)\cdot(0+x)\cdot z$$

Exercise 2. *1) Let the natural number $\mathbb{N}$ be our universe. We start by defining the property $P(q)$: q is even. We define the relation R on $\mathbb{N}$ as follows: $\forall_{n,m}$ $(n,m) \in R$ iff $P(n+m)$.*

a) Show that R is an equivalence relation.

b) Can you determine how many equivalence classes the relation R has?

2) Given the set $F = \{a, b, c, d, e\}$ with an ordering described by the diagram



Write down all subsets of F in which the element c is a minimal element.

Exercise 3. *a) Let G be a planar graph with eight vertices. What is the maximal number of edges possible in G?*

b) Let G be a finite graph. Can G have a subgraph H (H is supposed to be different from G) so that G and H are isomorphic?

c) Consider the graph G with $V(G) := \{a, b, c, d, e, f\}$ and $E(G) := \{\{a, c\}, \{a, d\}, \{a, e\}, \{b, e\}, \{b, f\}\}$.

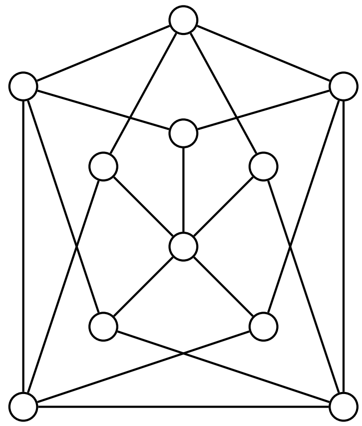
i) Determine $G - a$

ii) Find the connected components of $G - a$.

Exercise 4. *Let G be a planar graph with fewer than 12 vertices. Show that G has a vertex of degree at most 4. (Hint: Recall the formula $|E| \leq 3|V| - 6$.)*

Exercise 5. *Show that the following graph is not planar.*

Date: April 2, 2020.



(Hint: Notice that, in the case that the graph is planar, it contains no triangles. Then, any face would be bordered by at least 4 edges)

Exercise 6. *Use induction to show that for all natural numbers:* $4\sum_{k=1}^{n}(k^2+2k)(k+4) = (n^2+n)(n+4)(n+5)$.

Exercise 7. *Use the laws of logic to simplify the statement:* $\big(p \vee (p \wedge q) \vee (p \wedge q \wedge \neg r)\big) \wedge \big((p \wedge r \wedge t) \vee t\big)$

Exercise 8. *Lewis–Zax (page 67): exercise 6.1.*

Exercise 9. *Use the binomial theorem* $(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}$ *to compute the number*

$$\sum_{i=0}^{27}\binom{27}{i}(-3)^{2i+1}$$

Exercise 10. *Draw the state diagram* $D(M)$ *of the automaton* M *with states* $S := \{s_0, s_1, s_2\}$, *accepting states* $Y := \{s_0\}$, *input alphabet* $I := \{a, b\}$, *described in the state table* $T(M)$:

	ν	
	a	b
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_2	s_2

Exercise 11. *Draw the state diagram* $D(M)$ *of the automaton* M *with states* $S := \{s_0, s_1, s_2\}$, *accepting states* $Y := \{s_0, s_2\}$, *input alphabet* $I := \{a, b\}$, *described in the following state table* $T(M)$:

	ν	
	a	b
s_0	s_1	s_0
s_1	s_2	s_0
s_2	s_2	s_1

Which of the following input words are accepted by M *and which are not accepted by* M*?*

1) bbaab

2) abbab

3) aabbb

4) babaab
5) aaabbb

Exercise 12. *Let* $\Sigma := \{a, b\}$ *and define the language* $L := \{a^m b^n \mid m, n > 0\}$. *Construct an automaton* A' *which will accept the language* $L(A')$.

Exercise 13. a) *Draw the table for the automaton* A *in Fig. 1.*
b) *Find the language* $L(A)$ *accepted by the automaton* A *in Fig. 1.*

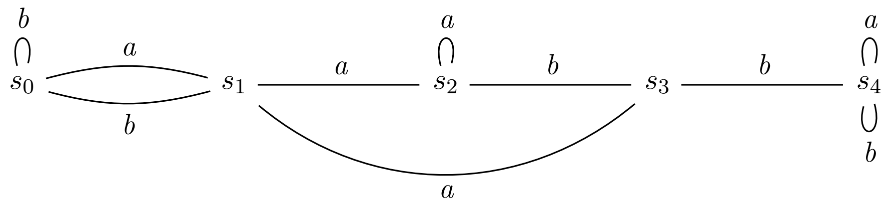


Figure 1. The automaton A.

Exercise 14. *Lewis–Zax (page 197/198): exercise 19.1, 19.2 a),b), 19.4.*

Exercise 15. *Draw the state diagram of the finite state machine* N *corresponding to the transition table*

N	ν		ω	
	0	1	0	1
s_0	s_0	s_1	0	0
s_1	s_1	s_2	0	0
s_2	s_2	s_3	0	0
s_3	s_3	s_4	0	0
s_4	s_4	s_5	0	0
s_5	s_5	s_0	0	1

What is the output corresponding to the input sequence 0110111011*?*

Exercise 16. *Given two states* s_0, s_1 *(*s_0 *being the starting state), complete the following diagram by adding arrows:*

s_0 s_1

so that it becomes a state diagram of the finite state machine M*, which is supposed to recognise (with an output* 1*) every* 0 *appearing in an input string* x *that is preceded by another* 0.