

MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2019

EXERCISE SET 2

- Exercise 1.** (Grimaldi, 5. ed., Exercises 2.3) *Exercise 1: b)*
- Exercise 2.** (Grimaldi, 5. ed., Exercises 2.3) *Exercise 2: d)*
- Exercise 3.** (Grimaldi, 5. ed., Exercises 2.3) *Exercise 10: c), f)*
- Exercise 4.** (Grimaldi, 5. ed., Exercises 2.3) *Exercise 13: a), b) iii), v)*
- Exercise 5.** (Grimaldi, 5. ed., Exercises 2.4) *Exercise 8: a), c), h)*
- Exercise 6.** (Grimaldi, 5. ed., Exercises 2.4) *Exercise 12: a) i), v), viii)*
- Exercise 7.** (Grimaldi, 5. ed., Exercises 2.5) *Exercise 9*
- Exercise 8.** (Grimaldi, 5. ed., Exercises 2.5) *Exercise 10*

★ **Exercise 9.** *Translate the following expressions into English and Norwegian and determine which of them are true statements (for numbers in \mathbb{R}):*

- (1) $\forall x \exists y (x > y \rightarrow x > y^2)$
- (2) $\forall x \forall y (x > y \rightarrow \exists z (x > z \wedge z > y))$
- (3) $\exists x \forall y \exists z ((x + y)z = 1)$
- (4) $\forall x (I(x) \rightarrow \exists y (I(y) \wedge (x < y)))$, $I(x)$: x is an integer.
- (5) $\neg \exists x \forall y (I(y) \rightarrow x > y)$, $I(x)$: x is an integer.

Note. *For the following exercise, recall that a prime number is a positive integer divisible only by itself and 1.*

★ **Exercise 10.** *Translate the following phrases English into statements (for numbers in \mathbb{R} ; use $I(x)$: x is an integer.):*

- (1) *No integer is both even and odd.*
- (2) *Not every real number is greater than an integer.*
- (3) *There are infinitely many primes. (Hint: this is equivalent to saying that for every natural number there is a greater prime.)*
- (4) *Every even integer greater than 2 is the sum of two primes. This is known as Goldbach's conjecture.*