MA0301
ELEMENTARY DISCRETE MATHEMATICS
NTNU, SPRING 2019

Exercise Set 10

NOTE: Problems marked with a $\star$ are mandatory. Their solutions must be included to get the set approved.

$\star$ **Exercise 1.** *i) How many ways are there to select 12 cars if Fords, BMWs, and Fiats are available?*

ii) How many if at least 1 car of each type must be selected?

iii) If at least 3 Fiats have to be selected?

iv) If at least 1 car of each type and at least 4 BMWs have to be selected?

$\star$ **Exercise 2.** *How many distinct non-negative integer solutions does the equation*

$$x_1 + x_2 + x_3 + x_4 = 12$$

have? Hint: think of this problem as having 12 *objects to distribute to four distinct recipients.*

$\star$ **Exercise 3.** *We have 30 cars in a car dealership. 20 cars have radios, 8 cars have air conditioners and 25 cars have fuel injection. Note: 20 have at least two of these features and 6 have all three. a) How may cars have at least one of the features? b) How many have none of these features? c) How many have exactly one?*

$\star$ **Exercise 4.** *We have n finite sets $A_1, A_2, \dots, A_n$. Prove by induction over n:*

$$|\bigcup_{i=1}^{n} A_i| = \sum_{\substack{J \subset [n] \\ J \neq \emptyset}} (-1)^{|J|-1} |\bigcap_{j \in J} A_j|,$$

where $[n] := \{1, \dots, n\}$.

$\star$ **Exercise 5.** *(Grimaldi, 5. ed., Exercises 1.4, page 36/33): Exercise 28*

$\star$ **Exercise 6.** *Show by induction that for all natural numbers*

$$\sum_{k=1}^{m} (4k^3 + 24k^2 + 32k) = m(m+1)(m+4)(m+5).$$

$\star$ **Exercise 7.** (Grimaldi, 5. ed., Exercises 15.4, page 741/815) *Exercise 4*

$\star$ **Exercise 8.** *Let A, B be arbitrary sets. Use the laws of set theory to show that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.*

$\star$ **Exercise 9.** *Show that*

$$\big((s \wedge \neg p) \to r\big) \Leftrightarrow \big(s \to (p \vee r)\big).$$

Date: March 15, 2019.

Exercise 10. *Let* $X := \{1, 2, 3, \ldots, 9, 10\}$*. Decide whether the following sentences are statements (and determine its truth value) or propositional functions (and determine its truth set).*

a) $\forall x \in X\ \exists y \in X\ (x + y < 14)$

b) $\forall y \in X\ (x + y < 14)$

c) $\forall x \in X\ \forall y \in X\ (x + y < 14)$

d) $\exists y \in X\ (x + y < 14)$

Exercise 11. *Find the appropriate values of* n_0 *such that* $n^2 - 6n + 8 \geq 0$*. Then show that the statement is true for all* $n \geq n_0$*.*

Exercise 12. *Which of the following sets are equal?*

$A_1 := \{x \mid x \in \mathbb{N}, x < 3\}$, $A_2 := \{x \mid x^2 = 4x - 3\}$, $A_3 := \{x \mid x \in \mathbb{N}, x \text{ odd}, x < 5\}$, $A_4 := \{x \mid x^2 = 3x - 2\}$, $A_5 := \{1, 2\}$, $A_6 := \{1, 1, 3\}$. $A_7 := \{1, 2, 1\}$, $A_8 := \{3, 1\}$,

Exercise 13. *Show that for any sets* A, B, C

$$A - (B \cap C) = (A - B) \cup (A - C)$$