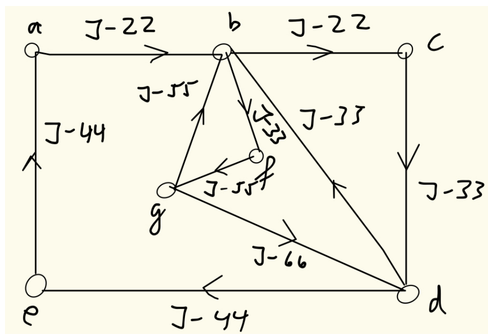
MA0301 ELEMENTARY DISCRETE MATHEMATICS
SPRING 2017

1. Homework Set 11 – Solutions

Exercise 1. *Grimaldi's book (5. ed., Exercises 11.1, page 519): solve* **Ex. 7**

Solution 1. a)



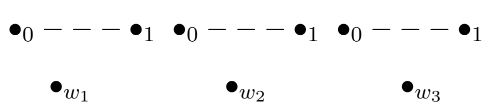
b) $\{(g,d),(d,e),(e,a)\}$ $\{(g,b),(b,c),(c,d),(d,e),(e,a)\}$
c) Two segments need to be blocked/deleted: one of $\{(b,c),(c,d)\}$ and one of $\{(b,f),(f,g),(g,d)\}$
d) no
e) Travel: $\{(c,d),(d,e),(e,a),(a,b),(b,f),(f,g)\}$
f) Travel: $\{(g,b),(b,f),(f,g),(g,d),(d,b),(b,c),(c,d),(d,e),(e,a),(a,b)\}$

Exercise 2. *Grimaldi's book (5. ed., Exercises 11.1, page 520): solve* **Ex. 15**

Solution 2. a_n denotes the number of closed $v-v$ walks of length n (=number of edges): $a_1 = 1$, $a_2 = 2$. For general $n > 2$ we find $a_n = a_{n-1} + a_{n-2}$ corresponding respectively to walks with the last segment being either the loop $\{v,v\}$ or the edge $\{v,w\}$. Hence, $a_n = F_{n+1}$, the n+1st Fibonacci number.

Exercise 3. *Grimaldi's book (5. ed., Exercises 11.1, page 520): solve* **Ex. 16**

Solution 3. a) There are two more:



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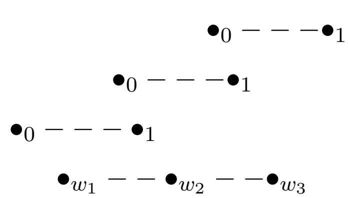

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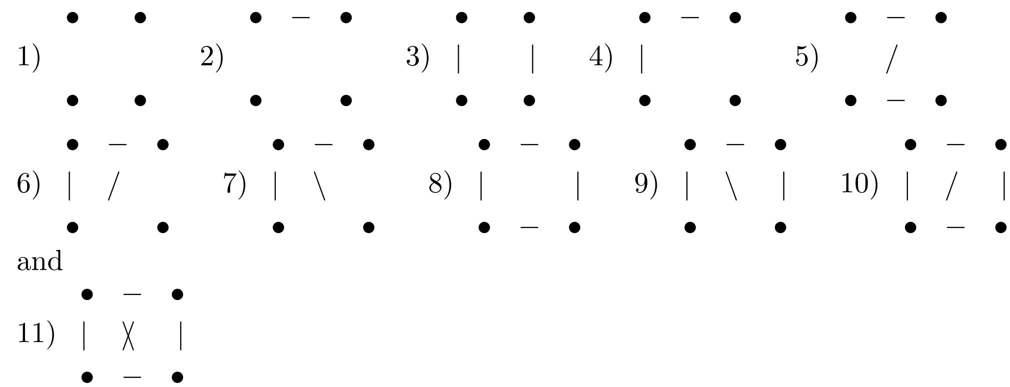
b) 14

c) Catalan number many: $C_n = \frac{1}{n+1}\binom{2n}{n}$, $n > 0$

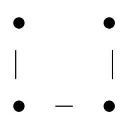
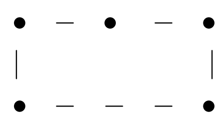
Exercise 4. *Grimaldi's book (5. ed., Exercises 11.2, page 529): solve* **Ex. 6**

Solution 4. You should find 11 loop-free undirected graphs with 4 vertices, which are nonisomorphic. Of those, 6 are connected.



Exercise 5. *Grimaldi's book (5. ed., Exercises 11.2, page 529): solve* **Ex. 12**

Solution 5. a) First, check that the graph K_n has $\binom{n}{2}$ edges. Then, for G with e_1 edges and $\overline{G}$ with e_2 edges, we have by definition that $e_1 + e_2 = \binom{n}{2}$. If G is self-complementary we have $e_1 = e_2$ from which we deduce that $2e_1 = \binom{n}{2}$. Hence, G must have $e_1 = n(n-1)/4$ edges.

b) $G_1 =$  $G_2 =$ 

c) Part a) implies for a self-complementary graph G that 4 must divide $n(n-1)$. Hence, either n or $n-1$ is odd (and the other even). So, if n is even (and $n-1$ odd), then 4 divides n and we must have $n = 4k$ for k a positive integer. If $n-1$ is even (and n odd), then 4 divides $n-1$ and we must have $n = 4k+1$ for k a positive integer.