MIDTERM EXAM MA0301 SPRING 2017

Exercise 1. *Determine all combinations (possibly none) of truth values for p, q and r for which the statement*

$$(p \to q) \vee (q \to r)$$

is false.

Exercise 2. *Let*

$$A = \{x \in \mathbb{N} \mid x \text{ is even}\},$$
$$B = \{x \in \mathbb{Z} \mid x \text{ is divisible by } 10^8\},$$
$$C = \{x \in \mathbb{Z} \mid 4x - 2 = 20\}.$$

Which of the following are true?

(a) $A \subseteq B$
(b) $B \subseteq A$
(c) $C \subseteq A$
(d) $C \subseteq \emptyset$.

Exercise 3. *Let F_i be the ith Fibonacci number, defined recursively by $F_0 = 0$, $F_1 = 1$ and $F_{n+2} = F_n + F_{n+1}$ for $n \geq 0$. Use induction to prove that no two consecutive Fibonacci numbers are even. That is, there is no $i \geq 0$ such that both F_i and F_{i+1} are even.*

Exercise 4. *Let $A = \{1, 2, 3, 4, 5, 6\}$ and let $\mathcal{R}$ be the relation on A given by $a\mathcal{R}b$ if $b \geq 2a$ or $b = a$. Draw the Hasse diagram for the poset $(A, \mathcal{R})$. What are the greatest lower bound and least upper bound of $\{2, 3\}$?*

Exercise 5. *Give three examples of relations $\mathcal{R}$ on $\{a, b, c\}$ such that $\mathcal{R}$ is reflexive and transitive, and $a\mathcal{R}b$ holds.*

Exercise 6. *Let $\{(x, y) \mid x, y \in \mathbb{Z}, (x^2 + x)/3 = y\}$ be a relation from $\mathbb{Z}$ to $\mathbb{Z}$. Is this relation a function from $\mathbb{Z}$ to $\mathbb{Z}$?*

Exercise 7. *Prove the identity*

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$

Exercise 8. *11 children want to play football. They decide to choose one referee, and divide the remaining ten into two teams of five players each. In how many different ways can they do this?*

Exercise 9. *How many surjective functions are there from $\{1, 2, \ldots, n+1\}$ to $\{1, 2, \ldots, n\}$?*