NTNU
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Department of Mathematical Sciences

Examination paper for
MA0301 Elementary discrete mathematics

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Informasjon om trykking av eksamensoppgave	
Originalen er:	
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Date Signature

Exercise 1: 15 points

Exercise 2: 10 points

Exercise 3: 15 points

Exercise 4: 15 points

Exercise 5: 20 points

Exercise 6: 15 points

Exercise 7: 10 points **Total: 100 points**

Problem 1 **Sets** (15 points) Use only the laws of set theory to prove the following statements for arbitrary sets A, B, C.

1. **(7 points)**
$$\text{If} \quad (A \cup B) \subseteq (A \cap B) \quad \text{then} \quad A = B.$$
2. **(4 points)**
$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$
3. **(4 points)**
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Problem 2 **Logic** (10 points)

1. **(6 points)** Use the laws of logic to simplify:
$$\Big(p \vee (p \wedge q) \vee (p \wedge q \wedge \neg r)\Big) \wedge \Big((p \wedge r \wedge t) \vee t\Big)$$

2. **(4 points)** Use a truth table to show that:
$$\Big((a \wedge b) \longrightarrow c\Big) \Leftrightarrow \Big((a \longrightarrow c) \vee (b \longrightarrow c)\Big)$$

Problem 3 Equivalence relation (15 points)

1. **(3 points)** Write down the definition of an equivalence relation.

2. **(2 points)** Write down the definition of an equivalence class.

3. **(10 points)** Let $A := \{1, 2, 3\}$. Determine whether the following relations on A are equivalence relations. Give an argument in each case. If an equivalence relation is given determine the equivalence classes.
   - i) (5 points) $R_1 := \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (3, 1), (1, 3)\}$
   - ii) (5 points) $R_2 := \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2), (1, 3), (3, 1)\}$

Problem 4 Functions (15 points)

1. **(2 points)** Give the definition of a surjective (onto) function.

2. **(3 points)** Give the definition of an injective (one-to-one) function.

3. **(5 points)** Let $g : A \to B$ and $f : B \to C$ be two functions. Prove that if g and f are both injective, then $f \circ g : A \to C$ is injective.

4. **(5 points)** Define the function $f : \mathbb{N} \to \mathbb{N}$ by $f(n) := 2n$. Show that f is injective and that f is not surjective.

Problem 5 Induction (20 points)

1. **(5 points)** Show by induction that for all natural numbers
$$\sum_{k=1}^{n} k(k + 2)(k + 4) = \frac{1}{4}n(n + 1)(n + 4)(n + 5).$$

2. **(7 points)** Prove by induction that for all positive integers
$$2 + 6 + 10 + \cdots + (4n - 2) = 2n^2.$$

3. **(8 points)** Show by induction that $n^3 - n$ is divisible by 3 for any positive integer n. (Recall that a positive integer m is divisible by 3 provided that there exists a positive integer t so that $m = 3t$).

Problem 6 Finite state automata (15 points)

1. **(10 points)** Draw the state diagram $D(M)$ of the automaton M with states $S := \{s_0, s_1, s_2\}$, accepting states $Y := \{s_0, s_2\}$, input alphabet $I := \{a, b\}$, described in the following state table $T(M)$:

	ν	
	a	b
s_0	s_1	s_0
s_1	s_2	s_0
s_2	s_2	s_1

2. **(5 points)** Which of the following input words are accepted by M and which are not accepted by M?

   1) *bbaab*

   2) *abbab*

   3) *aabbb*

   4) *babaab*

   5) *aaabbb*

Problem 7 Graphs (10 points)

(10 points) Let G be an arbitrary finite connected planar graph with at least three vertices. Show that G contains at least one vertex of degree equal to or smaller than five.