Recap

§1.1 Rules of Sum & Product

Rule of Sum
- A task can be done either by method 1. or by method 2.

To perform M1. $\exists$ m ways
M2. $\exists$ n ways. } $\Rightarrow$ To achieve the task $\exists$ m+n ways

Rule of Prod
- A task needs to done by step 1 & step 2

To perform S1. $\exists$ n ways
S2. $\exists$ n ways } $\Rightarrow$ $\exists$ n·n ways

§1.2
- # permutations of size r for n distinct objects
$= P(n,r) = \frac{n!}{(n-r)!}$
- # linear arrangements of given n obj (= n_1 indist. obj of 1st type + n_2 - - - - - 2nd + ⋮ + n_m m-th)
$= \frac{n!}{n_1! \, n_2! \cdots n_m!}$
- # combinations of size r from a collection of n dist. obj.
$= C(n,r) = \frac{n!}{r!(n-r)!}$ MUST KNOW

§1.3
Thm Binomial Theorem
$(x+y)^n = \binom{n}{0}x^0y^n + \binom{n}{1}x^1y^{n-1} + \cdots + \binom{n}{n}x^ny^0$
$= \sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k}$
- Special case: $x=y=1$
$x=1, y=-1$

§1.4

- #combinations (with repetition) of size r from n obj
  $= \binom{n+r-1}{r}$ = # solutions $x_1 + \cdots + x_n = r$ with $x_i \in \mathbb{N}$ $= \mathbb{Z}^+ \cup \{0\}$.

§2.

Notation

$\neg$	Not
$\wedge$	and
$\vee$	or
$\underline{\vee}$	exclusive or. (not both)
$\rightarrow$	imply
$\leftrightarrow$	iff equiv.

Laws P58

Implication P78 Table 2.6

Notations $\forall$ $\exists$.

P86 Table 2.23

$\lim_{x \to 0} f(x) = 0$

e.g. $\forall \varepsilon > 0, \exists \delta > 0$ s.t.
if $|x| < \delta$, then $|f(x)| < \varepsilon$.

§3.

Notations: $\subset$ inclusion (possibly equal)

- Power set. $\mathcal{P}(A) = \{\text{subsets of } A\}$

  If $|A| = n$ then $|\mathcal{P}(A)| = 2^n$.

Cor / P135 / Thm 3.6.

- $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$

§4.

- $A \subset \mathbb{Z}^+ \Rightarrow \exists$ smallest element in A.

§4.1

- Mathematical Induction "MUST KNOW"

§4.2 Recursive def/rel.

e.g. Fibonaicci numbers

§5.

Notions: function $A \to B$
injection / surjection / bijection.
inverse / composition.

§5.3 NO PROOF

- Pigeonhole Principle

- Complexity of algorithm
  Big-Oh Forms 293 / Table 5.11
  (Special functions)
- Analysis of Algorithm : calculate ↑

§6. Language .... Finite State Machine ★
How to construct
: "Add states to dist. cases"

§7.
Notions Relation Partial order (poset)
Hasse diagram of

- Topological Sorting Alg.
- Equivalent Relations / Partitions
  e.g. (mod. n) on $\mathbb{Z}$
  $x \sim y$ if $n \mid x-y$ $x \equiv y \pmod n$
  # equiv. classes = n (residue $\in \{0, 1, \dots, n\}$).
  $\mathbb{Z} = n\mathbb{Z} \cup \{n\mathbb{Z}+1\} \cup \cdots \cup \{n\mathbb{Z}+(n-1)\}$.

§11. Graph Theory.

Defs .... undirected/directed/multi

- isomorphism

Thm $G=(V,E)$ conn. (undirected multi)
Then $\exists$ Euler circuit $\Leftrightarrow 2 \mid \deg v$. $\forall v \in V$
$\exists$ Euler trail $\Leftrightarrow 2 \nmid \deg v$. holds for exactly 2 vertices $v \in V$.

- Directed version
  one more condition for $\exists$ Euler circuit:
  $id(v) = od(v)$ $(\Rightarrow 2 \mid \deg v.)$

(proof not required but need to know how to use)

Thm A graph is planar if $\nexists$ subgraph $\cong K_5$ or $\cong K_{3,3}$.

Thm planar: $v-e+r=2$

§12. Trees.

Depth/Breadth-First Search Alg.

- MergeSort Algorithm.

§13.

{ Shortest-Path Algo.
minimal Spanning tree Algo. { Kruskal's / Prim's