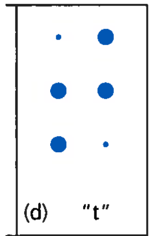
y listing all the se-
letters a, b, c, d, e,

;e, Diane decides to
er sister Ann Marie
an Diane make her

12) d) $\binom{15}{10}$

a lowercase letter,
ı by raising at least
hown in part (a) of
eled in this part of
figure the dots in
arrangement repre-
figure we have the
tively. The definite
ure, while part (f)
lly, the semicolon,
(g), where the dots




ve represent in the

ree raised dots?
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**6.** If $n$ is a positive integer and $n > 1$, prove that $\binom{n}{2} + \binom{n-1}{2}$ is a perfect square.

**7.** A committee of 12 is to be selected from 10 men and 10 women. In how many ways can the selection be carried out if (a) there are no restrictions? (b) there must be six men and six women? (c) there must be an even number of women? (d) there must be more women than men? (e) there must be at least eight men?

**8.** In how many ways can a gambler draw five cards from a standard deck and get (a) a flush (five cards of the same suit)? (b) four aces? (c) four of a kind? (d) three aces and two jacks? (e) three aces and a pair? (f) a full house (three of a kind and a pair)? (g) three of a kind? (h) two pairs?

**9.** How many bytes contain (a) exactly two 1's; (b) exactly four 1's; (c) exactly six 1's; (d) at least six 1's?

**10.** How many ways are there to pick a five-person basketball team from 12 possible players? How many selections include the weakest and the strongest players?

**11.** A student is to answer seven out of 10 questions on an examination. In how many ways can he make his selection if (a) there are no restrictions? (b) he must answer the first two questions? (c) he must answer at least four of the first six questions?

**12.** In how many ways can 12 different books be distributed among four children so that (a) each child gets three books? (b) the two oldest children get four books each and the two youngest get two books each?

**13.** How many arrangements of the letters in MISSISSIPPI have no consecutive S's?

**14.** A gym coach must select 11 seniors to play on a football team. If he can make his selection in 12,376 ways, how many seniors are eligible to play?

**15. a)** Fifteen points, no three of which are collinear, are given on a plane. How many lines do they determine?

**b)** Twenty-five points, no four of which are coplanar, are given in space. How many triangles do they determine? How many planes? How many tetrahedra (pyramidlike solids with four triangular faces)?

**16.** Determine the value of each of the following summations.

**a)** $\sum_{i=1}^{6}(i^2+1)$ **b)** $\sum_{j=-2}^{2}(j^3-1)$ **c)** $\sum_{i=0}^{10}[1+(-1)^i]$

**d)** $\sum_{k=n}^{2n}(-1)^k$, where $n$ is an odd positive integer

**e)** $\sum_{i=1}^{6} i(-1)^i$

**17.** Express each of the following using the summation (or Sigma) notation. In parts (a), (d), and (e), $n$ denotes a positive integer.

**a)** $\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots+\frac{1}{n!},\ n \geq 2$

**b)** $1+4+9+16+25+36+49$

**c)** $1^3-2^3+3^3-4^3+5^3-6^3+7^3$

**d)** $\frac{1}{n}+\frac{2}{n+1}+\frac{3}{n+2}+\cdots+\frac{n+1}{2n}$

**e)** $n-\left(\frac{n+1}{2!}\right)+\left(\frac{n+2}{4!}\right)-\left(\frac{n+3}{6!}\right)+\cdots+(-1)^n\left(\frac{2n}{(2n)!}\right)$

**18.** For the strings of length 10 in Example 1.24, how many have (a) four 0's, three 1's, and three 2's; (b) at least eight 1's; (c) weight 4?

**19.** Consider the collection of all strings of length 10 made up from the alphabet 0, 1, 2, and 3. How many of these strings have weight 3? How many have weight 4? How many have even weight?

**20.** In the three parts of Fig. 1.8, eight points are equally spaced and marked on the circumference of a given circle.

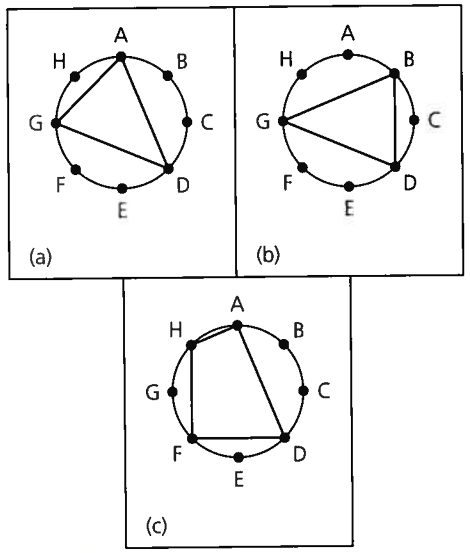


**Figure 1.8**

**a)** For parts (a) and (b) of Fig. 1.8 we have two different (though congruent) triangles. These two triangles (distinguished by their vertices) result from two selections of size 3 from the vertices A, B, C, D, E, F, G, H. How many different (whether congruent or not) triangles can we inscribe in the circle in this way?

**b)** How many different quadrilaterals can we inscribe in the circle, using the marked vertices? [One such quadrilateral appears in part (c) of Fig. 1.8.]

**c)** How many different polygons of three or more sides can we inscribe in the given circle by using three or more of the marked vertices?



**21.** How many triangles are determined by the vertices of a regular polygon of $n$ sides? How many if no side of the polygon is to be a side of any triangle?

**22. a)** In the complete expansion of $(a+b+c+d)\cdot(e+f+g+h)(u+v+w+x+y+z)$ one obtains the sum of terms such as $agw$, $cfx$, and $dgv$. How many such terms appear in this complete expansion?

**b)** Which of the following terms do *not* appear in the complete expansion from part (a)?

**i)** $afx$ **ii)** $bvx$ **iii)** $chz$
**iv)** $cgw$ **v)** $egu$ **vi)** $dfz$

**23.** Determine the coefficient of $x^9y^3$ in the expansions of (a) $(x+y)^{12}$, (b) $(x+2y)^{12}$, and (c) $(2x-3y)^{12}$.

**24.** Complete the details in the proof of the multinomial theorem.

**25.** Determine the coefficient of

**a)** $xyz^2$ in $(x+y+z)^4$

**b)** $xyz^2$ in $(w+x+y+z)^4$

**c)** $xyz^2$ in $(2x-y-z)^4$

**d)** $xyz^{-2}$ in $(x-2y+3z^{-1})^4$

**e)** $w^3x^2yz^2$ in $(2w-x+3y-2z)^8$

**26.** Find the coefficient of $w^2x^2y^2z^2$ in the expansion of (a) $(w+x+y+z+1)^{10}$, (b) $(2w-x+3y+z-2)^{12}$, and (c) $(v+w-2x+y+5z+3)^{12}$.

**27.** Determine the sum of all the coefficients in the expansions of

**a)** $(x+y)^3$ **b)** $(x+y)^{10}$ **c)** $(x+y+z)^{10}$

**d)** $(w+x+y+z)^5$

**e)** $(2s-3t+5u+6v-11w+3x+2y)^{10}$

**28.** For any positive integer $n$ determine

**a)** $\sum_{i=0}^{n}\frac{1}{i!(n-i)!}$ **b)** $\sum_{i=0}^{n}\frac{(-1)^i}{i!(n-i)!}$

**29.** Show that for all positive integers $m$ and $n$,

$$n\binom{m+n}{m}=(m+1)\binom{m+n}{m+1}.$$

**30.** With $n$ a positive integer, evaluate the sum

$$\binom{n}{0}+2\binom{n}{1}+2^2\binom{n}{2}+\cdots+2^k\binom{n}{k}+\cdots+2^n\binom{n}{n}.$$

**31.** For $x$ a real number and $n$ a positive integer, show that

**a)** $1=(1+x)^n-\binom{n}{1}x^1(1+x)^{n-1}$
$+\binom{n}{2}x^2(1+x)^{n-2}-\cdots+(-1)^n\binom{n}{n}x^n$

**b)** $1=(2+x)^n-\binom{n}{1}(x+1)(2+x)^{n-1}$
$+\binom{n}{2}(x+1)^2(2+x)^{n-2}-\cdots+(-1)^n\binom{n}{n}(x+1)^n$