$|A \cap C| = |B \cap C| = 1^5 = 1$ and $|A \cap B \cap C| = 0$, so by the preceding formula there are $|\overline{A} \cap \overline{B} \cap \overline{C}| = 3^5 - 3 \cdot 2^5 + 3 \cdot 1^5 - 0 = 150$ ways the student can select his daily games during a school week and play each type of game at least once.

This example can be expressed in an equivalent distribution form, since we are seeking the number of ways to distribute five distinct objects (Monday, Tuesday, . . . , Friday) among three distinct containers (the computer games) with no container left empty. More will be said about this in Chapter 5.

EXERCISES 3.3

1. During freshman orientation at a small liberal arts college, two showings of the latest James Bond movie were presented. Among the 600 freshmen, 80 attended the first showing and 125 attended the second showing, while 450 didn't make it to either showing. How many of the 600 freshmen attended twice?

2. A manufacturer of 2000 automobile batteries is concerned about defective terminals and defective plates. If 1920 of her batteries have neither defect, 60 have defective plates, and 20 have both defects, how many batteries have defective terminals?

3. A binary string of length 12 is made up of 12 bits (that is, 12 symbols, each of which is a 0 or a 1). How many such strings either start with three 1's or end in four 0's?

4. Determine $|A \cup B \cup C|$ when $|A| = 50$, $|B| = 500$, and $|C| = 5000$, if (a) $A \subseteq B \subseteq C$; (b) $A \cap B = A \cap C = B \cap C = \emptyset$; and (c) $|A \cap B| = |A \cap C| = |B \cap C| = 3$ and $|A \cap B \cap C| = 1$.

5. How many permutations of the digits 0, 1, 2, . . . , 9 either start with a 3 or end with a 7?

6. A professor has two dozen introductory textbooks on computer science and is concerned about their coverage of the topics (A) compilers, (B) data structures, and (C) operating systems.

The following data are the numbers of books that contain material on these topics:

$\vert A\vert = 8$	$\vert B\vert = 13$	$\vert C\vert = 13$
$\vert A \cap B\vert = 5$	$\vert A \cap C\vert = 3$	$\vert B \cap C\vert = 6$
$\vert A \cap B \cap C\vert = 2$		

(a) How many of the textbooks include material on exactly one of these topics? (b) How many do not deal with any of the topics? (c) How many have no material on compilers?

7. How many permutations of the 26 different letters of the alphabet contain (a) either the pattern "OUT" or the pattern "DIG"? (b) neither the pattern "MAN" nor the pattern "ANT"?

8. A six-character variable name in a certain version of ANSI FORTRAN starts with a letter of the alphabet. Each of the other five characters can be either a letter or a digit. (Repetitions are allowed.) How many six-character variable names contain the pattern "FUN" or the pattern "TIP"?

9. How many arrangements of the letters in MISCELLANEOUS have no pair of consecutive identical letters?

10. How many arrangements of the letters in CHEMIST have H before E, or E before T, or T before M? (Here "before" means anywhere before, not just immediately before.)

3.4 A First Word on Probability

When one performs an *experiment* such as tossing a single fair coin, rolling a single fair die, or selecting two students at random from a class of 20 to work on a project, a set of all possible outcomes for each situation is called a *sample space*. Consequently, {H, T} serves as a sample space for the first experiment mentioned and {1, 2, 3, 4, 5, 6} is a sample space for the roll of a single fair die. Moreover, $\{\{a_i, a_j\} | 1 \le i \le 20, 1 \le j \le 20, i \ne j\}$ can be used for the last experiment, with a_i denoting the ith student, for each $1 \le i \le 20$.

In dealing with the sample space $\mathscr{S} = \{1, 2, 3, 4, 5, 6\}$ for the roll of a single fair die, we feel that each of the six possible outcomes has the *same*, or *equal, likelihood* of occurrence. Using this assumption of equal likelihood, we shall start our study of probability theory with a definition for probability that was first given by the French mathematician Pierre-Simon de Laplace (1749–1827) in his *Analytic Theory of Probability*.

Under the assumption of equal likelihood, let $\mathscr{S}$ be the sample space for an expe
$\mathscr{E}$. Each subset A of $\mathscr{S}$, including the empty subset, is called an *event*. Each elem
$\mathscr{S}$ determines an *outcome*, so if $|\mathscr{S}| = n$ and $a \in \mathscr{S}$, $A \subseteq \mathscr{S}$, then

$Pr(\{a\})$ = *The probability that* $\{a\}$ *(or, a) occurs* $= \frac{|\{a\}|}{\mathscr{S}} = \frac{1}{n}$, and

$Pr(A)$ = *The probability that A occurs* $= \frac{|A|}{|\mathscr{S}|} = \frac{|A|}{n}$.

[*Note:* We often write $Pr(a)$ for $Pr(\{a\})$.]

We demonstrate these ideas in the following four examples.

EXAMPLE 3.28

When Daphne tosses a fair coin, what is the probability she gets a head? Here the
space $\mathscr{S} = \{H, T\}$ with $A = \{H\}$ and we find that

$$Pr(A) = \frac{|A|}{|\mathscr{S}|} = \frac{1}{2}.$$

EXAMPLE 3.29

If Dillon rolls a fair die, what is the probability he gets (a) a 5 or a 6, (b) an even n
For either part the sample space $\mathscr{S} = \{1, 2, 3, 4, 5, 6\}$. In part (a) we have event A
and $Pr(A) = \frac{|A|}{|\mathscr{S}|} = \frac{2}{6} = \frac{1}{3}$. For part (b) we consider event $B = \{2, 4, 6\}$ and f
$Pr(B) = \frac{|B|}{|\mathscr{S}|} = \frac{3}{6} = \frac{1}{2}$.

Furthermore we also notice here that

i) $Pr(\mathscr{S}) = \frac{|\mathscr{S}|}{|\mathscr{S}|} = \frac{6}{6} = 1$—after all, the occurrence of the event $\mathscr{S}$ is a certain

ii) $Pr(\overline{A}) = Pr(\{1, 2, 3, 4\}) = \frac{|\overline{A}|}{\mathscr{S}} = \frac{4}{6} = \frac{2}{3} = 1 - \frac{1}{3} = 1 - Pr(A)$.

EXAMPLE 3.30

There are 20 students enrolled in Mrs. Arnold's fourth-grade class. Hence, if she
select two of her students, at random, to take care of the class rabbit, she may m
selection in $\binom{20}{2} = 190$ ways, so $|\mathscr{S}| = 190$.

Now suppose that Kyle and Kody are two of the 20 students in the class and
be the event that Kyle is one of the students selected and B be the event that the s
includes Kody. Consequently, upon choosing the students, at random, the probabi
Mrs. Arnold selects

a) both Kyle and Kody is $Pr(A \cap B) = \binom{2}{2}/\binom{20}{2} = 1/190$;

b) neither Kyle nor Kody is $Pr(\overline{A} \cap \overline{B}) = \binom{18}{2}/\binom{20}{2} = 153/190$;

c) Kyle but not Kody is $Pr(A \cap \overline{B}) = \binom{1}{1}\binom{18}{1}/\binom{20}{2} = 18/190 = 9/95$.

EXAMPLE 3.31

Consider drawing five cards from a standard deck of 52 cards. This can be done i
2,598,960 ways. Now suppose that Tanya draws five cards, at random, from a s
deck. What is the probability she gets (a) three aces and two jacks; (b) three aces an
(c) a full house (that is, three of one kind and a pair)?