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Department of Mathematical Sciences

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EXAM IN MA0301 ELEMENTARY DISCRETE MATHEMATICS

English
Saturday, June 4, 2011
Time: 09:00-13:00
No printed or hand-written material is allowed during the exam.
An approved, simple calculator is allowed.

All problems have equal weight. Show your work.

Problem 1 How many words can be made from five different letters chosen from the word TRONDHEIM? How many of those words contain the letter O?

Problem 2 Find the coefficient of x^{10} in the expression $(3 + 4x^2)^{12}$.

Problem 3 Show that the following argument is valid:

$$\begin{array}{ll} & p \vee r \\ & p \to \neg q \\ & (r \wedge q) \to s \\ & q \\ \hline \therefore & s \end{array}$$

Problem 4 Define the integers a_n, where $n \in \mathbb{N}$, recursively by

$$\begin{aligned} a_0 &= 1 && \text{and} \\ a_n &= 3a_{n-1} - 1 && \text{for all } n \geq 1. \end{aligned}$$

Prove that

$$a_n = \frac{3^n + 1}{2}$$

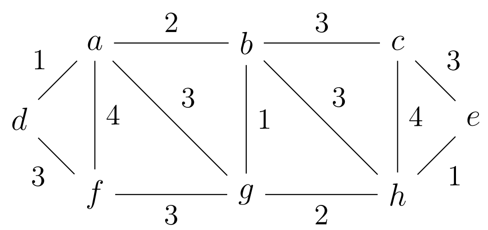
holds for all natural numbers n.

Problem 5 Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Find examples of functions $f : A \to B$ and $g : B \to A$ such that

a) the composition $g \circ f$ is bijective. In this case determine the inverse $(g \circ f)^{-1}$.

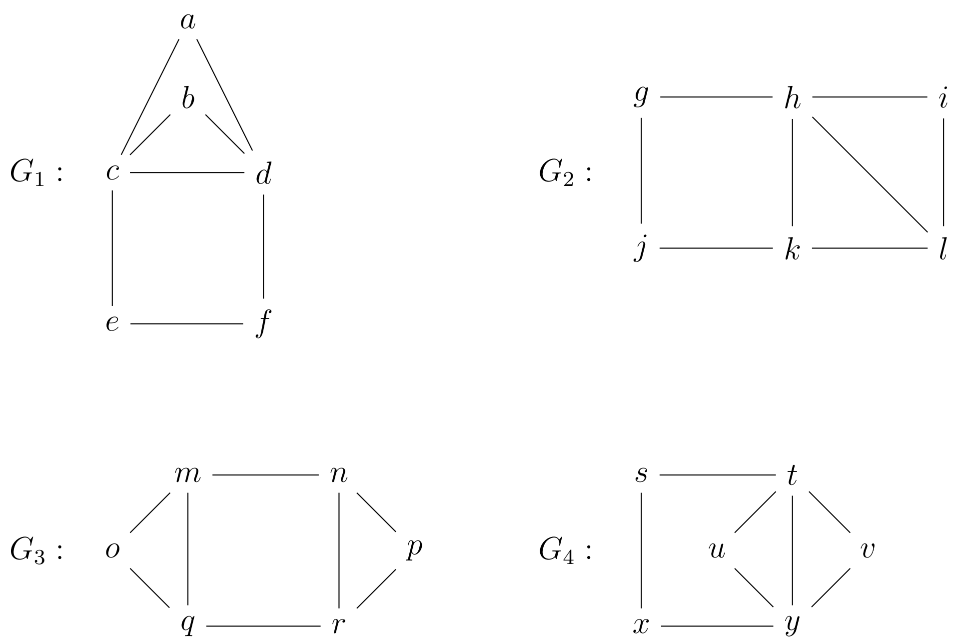
b) f is injective (one-to-one), g is surjective (onto) but $g \circ f$ is not bijective.

Problem 6 The following graph has four minimal spaning trees. Find them all.



Problem 7 Construct a finite state machine that recognizes the language $\{01\}^*\{11\}\{10\}^*$.

Problem 8 Consider the following four graphs. Which are isomorphic and which are not isomorphic?



Problem 9 Let F be the set of functions $f : \mathbb{Z} \to \mathbb{Z}$. Define the relations $\mathcal{R}$ and $\mathcal{S}$ on F by

$$f\mathcal{R}g \iff \forall x(f(x) \leq g(x)).$$
$$f\mathcal{S}g \iff \exists x(f(x) \leq g(x)).$$

a) Is the relation $\mathcal{R}$ a partial order?

b) Is the relation $\mathcal{S}$ a partial order?

Problem 10 Use Dijkstra's algorithm to find the shortest path from a to c in the following graph. Present all labels that are assigned to vertices as the algorithm runs.