Norwegian University of Science and Technology
Department of Mathematical Sciences

Contact during the exam:
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EXAM IN MA0301 ELEMENTARY DISCRETE MATHEMATICS

English
Tursday, May 28, 2009
Time: 0900-1300
No printed or hand-written material is allowed during the exam.
An approved, simple calculator is allowed.

All problems have equal weight. Show your work.

Problem 1 On an exam with ten yes/no-questions, the students must have at least four correct answers to pass, and at least nine correct answers to get top marks.

In how many different ways can the students answer the ten questions? How many of these correspond to a passing grade? How many correspond to a passing grade, but not to top marks?

Problem 2

a) Is $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$ a tautology?

b) Use the laws of logic to show that $p \leftrightarrow q$ og $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

c) Show that the conclusion $\neg p$ follows from the premises (i) $p \rightarrow q$, (ii) $\neg q \vee \neg r \vee \neg s$, (iii) $s \rightarrow r$ and (iv) s.

Problem 3 Give a finite state machine that recognizes all the strings in the language $\{1\}\{01\}^*\{01\} \cup \{0\}\{10\}^*\{1\}$.

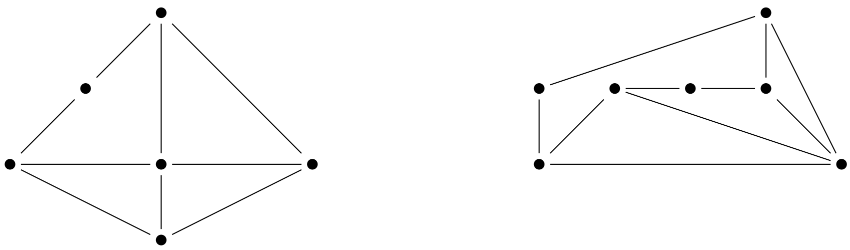
Problem 4

a) Show by induction on the number of vertices that the number of edges in the complete graph with n vertices is

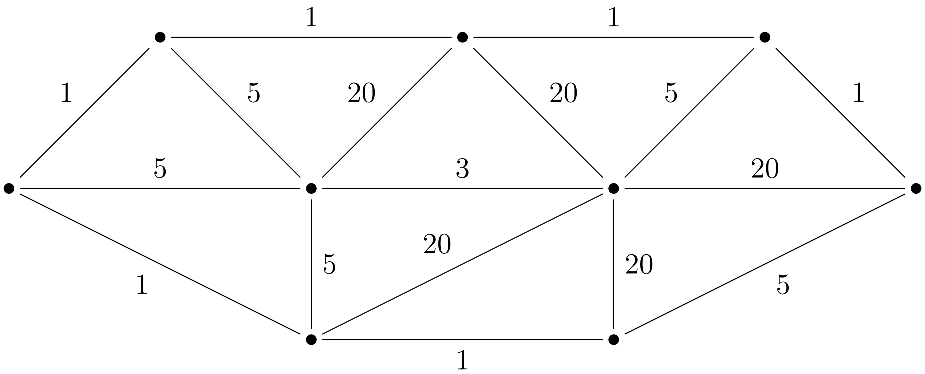
$$\sum_{i=1}^{n-1} i.$$

Note: When $n = 1$ we interpret the sum as 0.

b) Are the two graphs given here isomorphic? Homeomorphic?



c) What is a minimal spanning subtree? Use Kruskal's or Prim's algorithm to find a minimal spanning subtree for the weighted graph and the total weight for this subtree:



Problem 5 Let $\mathbb{N}$ be the natural numbers $\{0, 1, 2, \dots\}$ and let $\mathbb{Z}$ be the integers $\{\dots, -2, -1, 0, 1, 2, \dots\}$. Let $\sim$ be the relation on $\mathbb{N} \times \mathbb{N}$ given by

$$(a, b) \sim (c, d) \quad \Leftrightarrow \quad a + d = b + c.$$

a) Explain what an equivalence relation is.

Show that $\sim$ is an equivalence relation.

b) Explain what a bijection is.

Let S be the set of equivalence classes for $\sim$. We let $[(x, y)]$ denote the equivalence class that contains (x, y). Let $f : \mathbb{Z} \to S$ be a function given by

$$f(x) = \begin{cases} [(x, 0)] & x \geq 0 \\ [(0, -x)] & x < 0. \end{cases}$$

Show that f is a bijection. What is f^{-1}?