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MA0002 Brukerkurs i Matematikk B
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Solutions to exercise set 11

1 Use the properties for limits to compute the following limits

a)

$$\lim_{(x_1,x_2)\to(-1,1)} (2x_1x_2 + 3x_1^2)$$

$$\lim_{(x_1,x_2)\to(-1,1)} (2x_1x_2+3x_1^2) = \lim_{(x_1,x_2)\to(-1,1)} 2x_1x_2 + \lim_{(x_1,x_2)\to(-1,1)} 3x_1^2 = -2+3 = 1$$

b)

$$\lim_{(x_1,x_2)\to(1,1)} \frac{x_1x_2}{x_1^2 + x_2^2}$$

Since

$$\lim_{(x_1,x_2)\to(1,1)} (x_1^2 + x_2^2) \neq 0$$

we can compute

$$\lim_{(x_1,x_2)\to(1,1)} \frac{x_1x_2}{x_1^2 + x_2^2} = \frac{\lim_{(x_1,x_2)\to(1,1)} x_1x_2}{\lim_{(x_1,x_2)\to(1,1)}(x_1^2 + x_2^2)} = \frac{1}{2}$$

2 Show that

$$\lim_{(x_1,x_2)\to(0,0)} \frac{3x_1^2 - x_2^2}{x_1^2 + x_2^2}$$

does not exist by computing the limit along the positive x_1-axis and the positive x_2-axis.

We first compute along the x-axis

$$\lim_{(x_1,0)\to(0,0)} \frac{3x_1^2}{x_1^2} = \lim_{(x_1,0)\to(0,0)} 3 = 3.$$

Then we compute along the x_2-axis

$$\lim_{(0,x_2)\to(0,0)} \frac{-x_2^2}{x_2^2} = \lim_{(0,x_2)\to(0,0)} -1 = -1$$

Since these are not equal, the limit does not exists.

3 a) Compute

$$\lim_{(x_1,x_2)\to(0,0)} \frac{3x_1x_2}{x_1^2 + x_2^2}$$

along the lines $x_2 = mx_1$, for $m \neq 0$. What can you conclude?

When we compute the limit along the line $x_2 = mx_1$, it is the same as computing

$$\lim_{x_1 \to 0} \frac{3x_1(mx_1)}{x_1^2 + (mx_1)^2} = \left(\lim_{x_1 \to 0} \frac{x_1^2}{x_1^2}\right)\left(\lim_{x_1 \to 0} \frac{3m}{1+m^2}\right) = 1 \cdot \frac{3m}{1+m^2} = \frac{3m}{1+m^2}$$

If we set in for $m = 1$ we get $3/2$, and if we set in for $m = 2$ we get $6/5$, which means we can conclude that the limit does not exsist.

b) Show that

$$f(x_1, x_2) = \begin{cases} \frac{3x_1x_2}{x_1^2+x_2^3} & \text{for } (x_1, x_2) \neq (0,0) \\ 0 & \text{for } (x_1, x_2) = (0,0) \end{cases}$$

is discontinuous at $(0, 0)$.

From a, we see that the limit does not exsist at the point $(0, 0)$, which means that f is not continuous.

4 Find $\partial f / \partial x_1$ and $\partial f / \partial x$" for for the functions

a) $f(x_1, x_2) = 2x_1\sqrt{x_2} - \frac{3}{x_1x_2^2}$

$$\frac{\partial f}{\partial x_1} = 2\sqrt{x_2} + \frac{3}{x_1^2x_2^2}$$
$$\frac{\partial f}{\partial x_2} = \frac{x_1}{\sqrt{x_2}} + \frac{6}{x_1x_2^3}$$

b) $f(x_1, x_2) = e^{-x_1^2} \cos(x_1^2 - x_2^2)$

$$\frac{\partial f}{\partial x_1} = -2x_1e^{-x_2^2} \sin(x_1^2 - x_2^2)$$
$$\frac{\partial f}{\partial x_2} = -2x_2e^{-x_2^2} \cos(x_1^2 - x_2^2) + 2x_2e^{-x_2^2} \sin(x_1^2 - x_2^2)$$

5 Let

$$f(x_1, x_2) = 2x_1^3 - 3x_2x_1.$$

Compute $f_{x_1}(1, 2)$ and $f_{x_2}(1, 2)$, and give a representation of the tangent plane of f at $(1, 2)$.

$$f_{x_1}(1, 2) = (6 \cdot 1^2 - 6) = 0$$
$$f_{x_2}(1, 2) = -3 \cdot 1 = -3$$

To find the tangent plane, we also need to compute $f(1, 2) = -4$. So we have the tangent plane

$$L(x_1, x_2) = -4 - 3(x_2 + 3) = -4 - 3x_2 - 3$$

6 Let

$$\mathbf{f}(x_1, x_2) = \begin{bmatrix} 3x_1 - x_2^2 \\ 4x_2 \end{bmatrix}.$$

Find the linear approximation of $\mathbf{f}$ at $(-1, -2)$.

To compute the linear approximation of $\mathbf{f}$ we first compute the following

$$\mathbf{f}(-1, -2) = \begin{bmatrix} -7 \\ -8 \end{bmatrix},$$

$$(D\mathbf{f})(x_1, x_2) = \begin{bmatrix} 3 & -2x_2 \\ 0 & 4 \end{bmatrix},$$

$$(D\mathbf{f})(-1, -2) = \begin{bmatrix} 3 & 4 \\ 0 & 4 \end{bmatrix}$$

Then the linear approximation of $\mathbf{f}$ is

$$\mathbf{L}(x_1, x_2) = \begin{bmatrix} -7 \\ -8 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1 + 1 \\ x_2 + 2 \end{bmatrix} = \begin{bmatrix} 3x_1 + 4x_2 + 4 \\ 4x_2 \end{bmatrix}.$$