Norwegian University of Science and Technology
Department of Mathematical Sciences

MA0002 Mathematical Methods B
Spring 2023

Exercise set 3

1 Solve the pure-time differential equation:

$$\frac{dx}{dt} = \cos\left(2\pi(t-3)\right), \text{ where } x(3) = 1.$$

2 Suppose that the amount of phosphorus in a lake at time t, denoted by $P(t)$, follows the equation

$$\frac{dP}{dt} = 3t + 1 \quad \text{with } P(0) = 0.$$

Find the amount of phosphorus at time $t = 10$.

3 Solve the autonomous differential equations:

$$\frac{dx}{dt} = 1 - 3x, \text{ where } x(-1) = -2.$$

4 Assume that $W(t)$ denotes the amount of radioactive material in a substance at time t. Radioactive decay is then described by the differential equation

$$\frac{dW}{dt} = -\lambda W(t), \quad \text{with } W(0) = W_0.$$

(a) Find $W(t)$ by solving the differential equation.

(b) Assume that $W(0) = 123$gr, $W(5) = 20$gr and that time is measured in minutes. Find the decay constant λ and determine the half-life of the radioactive substance.

5 Use the partial fraction decomposition to solve

$$\frac{dy}{dx} = (y-1)(y-2), \text{ where } y_0 = 0 \text{ if } x_0 = 0.$$

6 Solve the differential equation:

$$\frac{du}{dt} = x^2y^2, \text{ where } y_0 = 1 \text{ if } x_0 = 1.$$