Norwegian University of Science
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Institutt for matematiske fag

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Exercise set 11

1 Use the properties for limits to compute the following limits

a)
$$\lim_{(x_1,x_2)\to(-1,1)} (2x_1x_2 + 3x_1^2)$$

b)
$$\lim_{(x_1,x_2)\to(1,1)} \frac{x_1x_2}{x_1^2 + x_2^2}$$

2 Show that
$$\lim_{(x_1,x_2)\to(0,0)} \frac{3x_1^2 - x_2^2}{x_1^2 + x_2^2}$$
does not exist by computing the limit along the positive x_1-axis and the positive x_2-axis.

3 a) Compute
$$\lim_{(x_1,x_2)\to(0,0)} \frac{3x_1x_2}{x_1^2 + x_2^2}$$
along the lines $x_2 = mx_1$, for $m \neq 0$. What can you conclude?

b) Show that
$$f(x_1, x_2) = \begin{cases} \frac{3x_1x_2}{x_1^2+x_2^3} & \text{for } (x_1, x_2) \neq (0, 0) \\ 0 & \text{for } (x_1, x_2) = (0, 0) \end{cases}$$
is discontinuous at $(0, 0)$.

4 Find $\partial f/\partial x_1$ and $\partial f/\partial x_»$ for for the functions

a) $f(x_1, x_2) = 2x_1\sqrt{x_2} - \frac{3}{x_1x_2^2}$

b) $f(x_1, x_2) = e^{-x_1^2}\cos(x_1^2 - x_2^2)$

5 Let
$$f(x_1, x_2) = 2x_1^3 - 3x_2x_1.$$
Compute $f_{x_1}(1, 2)$ and $f_{x_2}(1, 2)$, and give a representation of the tangent plane of f at $(1, 2)$.

6 Let

$$\mathbf{f}(x_1, x_2) = \begin{bmatrix} 3x_1 - x_2^2 \\ 4x_2 \end{bmatrix}.$$

Find the linear approximation of $\mathbf{f}$ at $(-1, -2)$.