Phase Plots of a System of DEs

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Phase Plots

When we deal with a system of the form

$$\frac{dy}{dt} = Ay,$$

the collection of all solution curves is called the **phase plot** of the system.

We are able to draw a qualitative sketch of the phase plot:

- When the eigenvalues λ_1, λ_2 are distinct and have opposite signs, we draw the two axes corresponding to their eigenvectors u and v.

Phase Plots

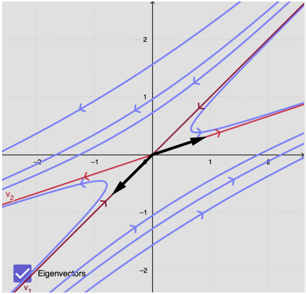
W show the phase plot for the system

$$\frac{dy}{dt} = Ay$$

with $A = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}$. A has the eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -1$ with eigenvectors $u = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Phase Plots

- For eigenvectors that correspond to a positive eigenvalue, the solution curves move away from the origin in the direction of the eigenvector.
- For eigenvectors that correspond to a negative eigenvalue, the solution curves move towards the origin in the direction of the eigenvector.



Phase Plots

What is a phase plot and what does it represent?

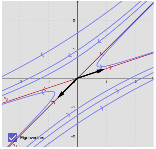
Suppose that we have a point that moves in the y_1y_2-plane, and its coordinate vector

$$y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$$

satisfies the DE

$$\frac{dy}{dt} = Ay.$$

If the point starts moving at time $t = 0$ from the point $\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix}$, then it will move along the curve/trajectory of the phase plot that goes through the point $\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix}$.



Phase Plots - Example

Sketch the phase plot for the system

$$\frac{dy}{dt} = Ay$$

with $A = \begin{pmatrix} 2 & -2 \\ 2 & -3 \end{pmatrix}$. A has the eigenvalues $\lambda_1 = -2$ and $\lambda_2 = 1$ with corresponding eigenvectors $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.