MA0002
Forelesning 2A/B

Eksempel

Finn: $\displaystyle\int_{1/2}^{1} \frac{1}{x^2} \cdot e^{1/x}\, dx = I$

$u = \frac{1}{x} = x^{-1}$ $\quad (= x^{u})$

$\frac{du}{dx} = (-1) \cdot x^{-2} = \frac{-1}{x^2}$

$\Rightarrow dx = -x^2\, du$

$$I = \int_{x=1/2}^{x=1} \frac{1}{x^2} \cdot e^{u} \cdot (-x^2) \cdot du = \int_{x=1/2}^{x=1} -e^{u}\, du$$

(a) $= \left[-e^{u}\right]_{x=1/2}^{x=1} = \left[-e^{1/x}\right]_{x=1/2}^{1}$

$= -e^{1} - (-e^{2}) = \underline{\underline{e^2 - e}}$

(b) $= \displaystyle\int_{u=2}^{u=1} -e^{u}\, du$

$= \left[-e^{u}\right]_{2}^{1} = \underline{\underline{e^2 - e}}$

x	u
$1/2$	2
1	1

Delvis Integrasjon

Påmin.: $(f(g(x)))' = f'(g(x)) \cdot g'(x)$

$$\Rightarrow \int f'(u) \cdot g'(x)\, dx = f(g(x)) + C$$

$u = g$

$$(u(x) \cdot v(x))' = u'(x) \cdot v(x) + u(x) \cdot v'(x)$$

$$\int (u(x) \cdot v(x))' - u'(x) \cdot v(x)\, dx = \int u(x) \cdot v'(x)\, dx$$

$$u(x) \cdot v(x) - \int u'(x) \cdot v(x)\, dx = \int u(x) \cdot v'(x)\, dx$$

Teorem Hvis $u(x)$ og $v(x)$ er deriv. funks, så

$$\int u(x) \cdot v'(x)\, dx = u(x) \cdot v(x) - \int u'(x) \cdot v(x)\, dx$$

Spesielt:
$$\int_a^b u \cdot v'\, dx = [u \cdot v]_a^b - \int_a^b u' \cdot v\, dx$$

Oppg: Finn: $\int \underbrace{x}_{v'} \cdot \underbrace{\ln x}_{u}\, dx = I$ $\qquad \ln|x| = \int \frac{1}{x}\, dx$

$$(\ln|x|)' = \frac{1}{x}$$

$u = \ln x \qquad v' = x$

$u' = \frac{1}{x} \qquad v = \frac{1}{2}x^2$

$$I = \underbrace{\frac{1}{2} \cdot x^2}_{v} \cdot \underbrace{\ln x}_{u} - \int \underbrace{\frac{1}{2} \cdot x^2}_{v} \cdot \underbrace{\frac{1}{x}}_{u'}\, dx$$

$$= \frac{1}{2} x^2 \cdot \ln x - \int \frac{x}{2}\, dx$$

$$= \frac{x^2}{2} \cdot \ln x - \frac{x^2}{4} + C$$

$I = \int \underbrace{e^x}_{u'} \cdot \underbrace{\cos x}_{v}\, dx$

$e^x = u$	$e^x = u'$
$v = \cos x$	$v' = -\sin x$

$$= e^x \cdot \cos x - \int e^x(-\sin x)\, dx$$

$$= e^x \cdot \cos x + \int e^x \cdot \sin x\, dx$$

$e^x = u$	$e^x = u'$
$v = \sin x$	$v' = \cos x$

$$I = e^x \cdot \cos x + \underbrace{\int e^x \cdot \sin x} \qquad u' = e^x \quad u = e^x \qquad v' = \cos x \quad v = \sin x$$

$$= e^x \cdot \cos x + \left[e^x \cdot \sin x - \underbrace{\int e^x \cdot \cos x \, dx}_{I} \right]$$

$$I = e^x \cdot \cos x + e^x \cdot \sin x - I \qquad | + I$$

$$2I = e^x \cdot \cos x + e^x \cdot \sin x \qquad | \div 2$$

$$I = \frac{e^x \cdot \cos x + e^x \cdot \sin x}{2}$$