Norwegian University of Science and Technology
Department of Mathematical Sciences

# Mathematical Methods B
# Spring 2022

**Exercise set 10**

**Vanskelighetsgrad: Easy (mere calculations)**

10.5.4 Let $f(x,y) = \ln(x \cdot y - x^2)$ with $x(t) = t^2$ and $y(t) = t$. Find the derivative $w'(5)$ for

$$w(t) = f(x(t), y(t)).$$

10.5.20 Find the gradient of

$$f(x,y) = x \cdot \left(x^2 - y^2\right)^{\frac{2}{3}}.$$

**Vanskelighetsgrad: Medium (Apply a theorem to reduce the amount of calculations)**

3 Given the function

$$f(x,y,z) = x \cdot y^2 + \sin(x \cdot z) + \ln\left(\frac{y-z}{y^2+y}\right)$$

with $x \in \mathbb{R}$, $y \in \mathbb{R}$, $y > 0$, and $z \in \mathbb{R}$, $z < y$.

Let $g(x,y,z) = \dfrac{\partial f(x,y,z)}{\partial y}$ and $h(x,y,z) = \dfrac{\partial g(x,y,z)}{\partial z}$. Find

$$\frac{\partial h(x,y,z)}{\partial x}.$$

**Hint:** All functions in this task are infinetely differentiable in an area around each point in $f$'s domain.

**Difficulty level: More challenging (Several aspects combined, recommended as preparation for the exam)**

4 Consider the functions

$$f(x,y) = x \cdot (1+y^2), \qquad g(x,y) = y \cdot (1+x^2).$$

The task is to find (approximate) a point $(x^*, y^*)$ in the $x$-$y$-planet, such that

$$f(x^*, y^*) = 1 \quad g(x^*, y^*) = 2\,.$$

Assume one knows that $(x^*, y^*)$ lies in proximity to $(a, b) = (0.2, 1.8)$. To find a better approximation one can use tandentplanes to $f$ and $g$ i $(a, b)$ and determine a point $(\bar{a}, \bar{b})$ where both approximations are zero.

(a) Calculte «how good» the approximation $(x^*, b^*) = (a, b)$ is already: Determine the values of $f$ and $g$ here.

(b) Find the approximations $z_f(x, y) \approx f(x, y)$, $z_g(x, y) = g(x, y)$ when you determine the tangentplanes at $(a, b)$

(c) Find the point $(\bar{a}, \bar{b})$ where $z_f = 1 and z_g = 2$.

(d) Is $(\bar{a}, \bar{b})$ actually a better approximation? Determine $f$ and $g$ at this point.

**Recommendation:** Use a calculator /computer and calculate 6 significant digits.

**Difficulty level: Challenge**

5 Here you are to create your own tasks. The task(s) should be such that:

(a) it involves several aspects from the chapter «multidimensional calculus»
(b) it's impossible to «Google» the solution
(c) there are some milestones so that one can check that one are on the right track

In addition you will create **suggested solution(s)** for your task(s). You may use the internet or literatyre for inspiration, but do remember to quote. For this task you may also choose to work in groups of up to 4 students. Only one person per group has to submit the task(s), but it is important that everyone in the group writes who they have collaborated with.

**Deadline:** Sunday, March 27, 2022 (digitally as a single pdf-file via Blackboard)