MA0002 Mathematical Methods
Spring 2022

Norwegian University of Science and Technology
Department of Mathematical Sciences

Exercise set 6

$\boxed{1}$ Explain with your own words in tasks (a) and (b). You may also use drawings.

(a) What is an eigenvector?

(b) What is an eigenvalue?

$\boxed{2}$ Find eigenvectors and eigenvalues to the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}.$$

$\boxed{3}$ Is $\boldsymbol{x} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ an eigenvector of the matrix $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ -2 & -2 \end{pmatrix}$? Why/why not?

$\boxed{4}$ Let $\mathbf{A}$ be an $n \times n$ matrix, and $\boldsymbol{x} \in \mathbb{R}^n$ an eigenvector of $\mathbf{A}$ with corresponding eigenvalue $\lambda \in \mathbb{R}$.

1. Which of the following statements are true, and which are false?
2. Why/why not?

(a) $\mathbf{A} \cdot \boldsymbol{x} = \lambda \cdot \mathbf{A}$

(b) $(\lambda \cdot \mathbf{I} - \mathbf{A}) \cdot \boldsymbol{x} = \mathbf{0}$

(c) $\mathbf{A} \cdot \boldsymbol{x} = \lambda \cdot \boldsymbol{x}$

(d) $\det(\lambda \cdot \mathbf{I} - \mathbf{A}) = 0$

(e) Multiplication with $\boldsymbol{x}$ scales $\mathbf{A}$ by a factor of λ

(f) Multiplication with $\mathbf{A}$ scales $\boldsymbol{x}$ by a factor of λ

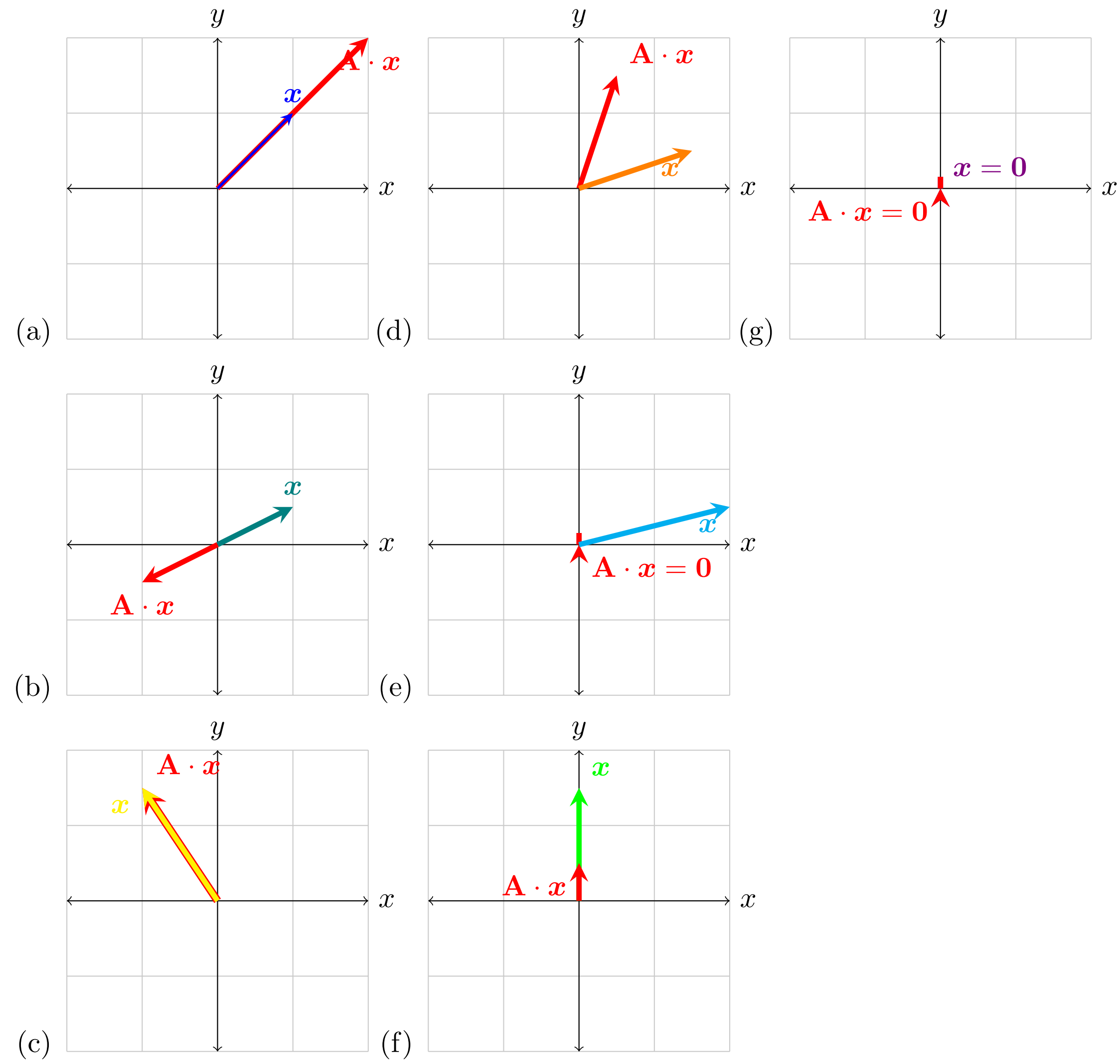
(g) $\mathbf{A} \cdot \boldsymbol{x}$ cannot be $\mathbf{0}$

(h) λ cannot be 0

(i) $\boldsymbol{x}$ cannot be $\mathbf{0}$

5 Let $\mathbf{A}$ be a 2×2-matrix and $\boldsymbol{x}$ a two-dimensional vector. In the cases (a) to (g) below, answer the following questions:

1. Is $\boldsymbol{x}$ an eigenvector of $\mathbf{A}$?
2. Why/why not?
3. In case that $\boldsymbol{x}$ *is* an eigenvector, what do you think is the corresponding eigenvalue $\lambda \in \mathbb{R}$?



6 Let

$$\mathbf{A} = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}.$$

(a) Show that $\boldsymbol{u}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\boldsymbol{u}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ are eigenvectors of A and that $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ are linearly independent

(b) Represent $\boldsymbol{x} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ as a linear combination of $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$.

(c) Use the results from (a) and (b) for å to calculate $\mathbf{A}^{20}\boldsymbol{x}$.

Deadline Sunday February 27th (digitally as a single pdf-file via Blackboard)