Norwegian University of Science and Technology
Department of Mathematical Sciences

Mathematical Methods B
Spring 2022

Exercise set 5

9.2.48 Let

$$\mathbf{A} = \begin{pmatrix} -1 & 1 \\ 2 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & -2 \\ 3 & 2 \end{pmatrix}.$$

Show that

$$(\mathbf{A} \cdot \mathbf{B})^{-1} = \mathbf{B}^{-1} \cdot \mathbf{A}^{-1}.$$

9.2.51 Let

$$\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 2 & -3 \end{pmatrix}$$

and

$$\boldsymbol{d} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}.$$

Find $\boldsymbol{x}$ such that $\mathbf{A} \cdot \boldsymbol{x} = \boldsymbol{d}$ by

(a) solving the associated system of linear equations and,

(b) using the inverse of $\mathbf{A}$.

9.2.57 Suppose that $\mathbf{A} = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix}$.

(a) Compute $\det \mathbf{A}$. Is $\mathbf{A}$ invertible?

(b) Suppose that $\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$.
Write $\mathbf{A} \cdot \boldsymbol{x} = \boldsymbol{b}$ as a system of linear equations.

(c) Show that if $\boldsymbol{b} = \begin{pmatrix} 3 \\ \frac{9}{2} \end{pmatrix}$, then $\mathbf{A} \cdot \boldsymbol{x} = \boldsymbol{b}$ has infinetely many solutions.
Graph the two straight lines associated with the corresponding system of linear equations, and explain why the system has infinetely many solutions.

(d) Find a column vector $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ so that $\mathbf{A} \cdot \boldsymbol{x} = \boldsymbol{b}$ has no solutions.

9.2.58 Suppose that

$$A = \begin{bmatrix} a & 8 \\ 2 & 4 \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \text{and} \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}.$$

(a) Show that when $a \neq 4$, $A\boldsymbol{x} = \boldsymbol{b}$ has exactly one solution.

(b) Suppose $a = 4$. Find conditions on b_1 and b_2 such that $A\boldsymbol{x} = \boldsymbol{b}$ has (i) infinetely many solutions and (ii) no solutions.

(c) Explain your results in (a) og (b) graphically.

5 Use Gauss-Jordan elimination to find the inverse matrix $\mathbf{A}^{-1}$ for

$$\mathbf{A} = \begin{pmatrix} 0 & 2 & 3 \\ 2 & -1 & -3 \\ -4 & 0 & 2 \end{pmatrix}.$$

From the exam spring 2013, does not have to be submitted Let

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 3 \\ -2 & 1 & -6 \\ 2 & 0 & 7 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 7 & 0 & -3 \\ 2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}.$$

Calculate the matrix products $\mathbf{A} \cdot \mathbf{B}$ and $\mathbf{B} \cdot \mathbf{A}$. What does this tell you about $\mathbf{A}$ and $\mathbf{B}$?

Deadline: Sunday, February 20th (digitally as a single pdf-file via Blackboard)