Norwegian University of Science
and Technology
Institutt for matematiske fag

# MA0002 Brukerkurs Matematikk B Vår 2022

**Oppgavesett 2**

1 Use polynomial division to write

$$f(x) := \frac{3x^3 + 5x - 2x^2 - 2}{x^2 + 1}$$

as a sum of a polynomial and a rational functjon $\frac{P}{Q}$ where $\deg P \leq \deg Q$. ($\deg P$ signifies the degree of the polynomial $P$.)

2 Evaluate the integral $I := \int f(x)\,\mathrm{d}x$ where

$$f(x) := \frac{4x^2 - x - 1}{(x+1)^2(x-3)}$$

using partial fraction decomposition.

3 Evaluate the integral

$$\int \frac{4}{(1-x)(1+x)^2}\,\mathrm{d}x.$$

4 Solve the initial value problem (IVP)

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \sqrt{3t+1}, \qquad s(0) = 1.$$

5 The task is to calculate

$$I = \int_0^{\pi/2} h(x)\,\mathrm{d}x$$

with

$$h(x) = \frac{(\sin x)^2}{(1 + \sin x + \cos x)^3}\,.$$

(a) To use the substitution $u = \tan(x/2)$ in the calculations, one first needs to express $\sin x$, $\cos x$ and $\mathrm{d}x$ in terms of $u$. Find these expressions. (**Hint:** Search for "universal substitution" or "Weierstrass substitution".)

(b) Use the above substitution to write $I$ as an integral in the variable $u$. **Hint:** The solution is given by $I = \int_0^1 \frac{u^2}{(u+1)^3}\,\mathrm{d}u$.

(c) Use the result from (b) to find $I$.

6 A lizard colony consists of individuals from three age groups: 0-year old, 1-year old and 2-year old. We only look at females in the colony. Usually, lizards do not have any offspring in their first year of their lives. A fifth of the 0-year old lizards survives to become one year old. The one-year old age group has on average two offspring. Half of the lizards survives their first year of life. As two-year olds, they have on average $b$ offspring, where $b$ is a given real number. When the observation of the colony started, there were 240 individuals in the first age group, 400 in the second, and 320 were two years old.

Calculate how many individuals of each age group are their (on average) after one, two and three years.

7 A population of algae in a lake is measured with equipment that reacts to signalling chemicals in the water which the algae excrete when they reproduce. As a result, it is only possible to measure the rate of change of the algae population, not the amount itself.

Let $m(t)$ be the amount of algae in that lake at time $t$ and assume that the rate of change was measured (and interpolated) as

$$m'(t) = \frac{\mathrm{d}m(t)}{\mathrm{d}t} = \frac{4 \cdot t}{5 + t^2}\,.$$

Assume furthermore that there was a measurement of the total algae amount at the beginning of the experiment. The result was

$$m(0) = 10.$$

Find the total amount of algae $m(5)$ under these assumptions.

**Hint:** Use the Fundamental Theorem of Calculus.

**Innleveringsfrist:** Søndag, 30. januar (digitalt som én pdf via Blackboard)