Norwegian University of Science
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Department of mathematical
sciences

MA0002 Mathematical Methods B Spring 2022

Exercise set 4

9.1.18 Laboratory mice are fed with a mixture of two foods that contain two essential nutrients. Food 1 contains 3 units of nutrient A and 2 units of nutrient B per ounce; food 2 contains 4 units of nutrient A and 5 units of nutrient B per ounce.

(a) In what proportion should you mix the food if the mice require nutrients A and B in equal amounts?

(b) Assume now that the mice require nutrients A and B in the ratio $1:2$. Is it possible to satisfy their dietary needs with the two foods available?

9.2.30 Suppose that $\mathbf{A}$ is an (l,p) matrix, $\mathbf{B}$ is an (m,q) matrix, and $\mathbf{C}$ is an (n,r) matrix. What can you say about l, m, n, p, q, and r if the products that follow are defined? State the size of the resulting matrix.

(a) $\mathbf{A}\cdot\mathbf{B}\cdot\mathbf{C}$

(b) $\mathbf{A}\cdot\mathbf{B}^\top\cdot\mathbf{C}$

(c) $\mathbf{B}\cdot\mathbf{A}\cdot\mathbf{C}^\top$

(d) $\mathbf{A}^\top\cdot\mathbf{C}\cdot\mathbf{B}^\top$

3 Let $A=\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\ -1 & 1\end{bmatrix}$.

(a) Find AB.

(b) Find BA.

4 Given the matrix

$$\mathbf{A}=\begin{pmatrix}a_{1,1} & a_{1,2}\\ a_{2,1} & a_{22}\end{pmatrix}.$$

Assume that $a_{1,1}\cdot a_{2,2}-a_{1,2}\cdot a_{2,1}\neq 0$.

(a) What is the inverse $\mathbf{A}^{-1}$?

(b) What can you say about $\mathbf{A}$ when $a_{1,1}\cdot a_{2,2}-a_{1,2}\cdot a_{2,1}=0$?

5 Let

$$B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}.$$

(a) Calculate B^2, B^3, B^4 og B^5.

(b) What can you say about B^k when (i) k is an even number (ii) k is an odd number?

6 (a) Let

$$\mathbf{A} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}.$$

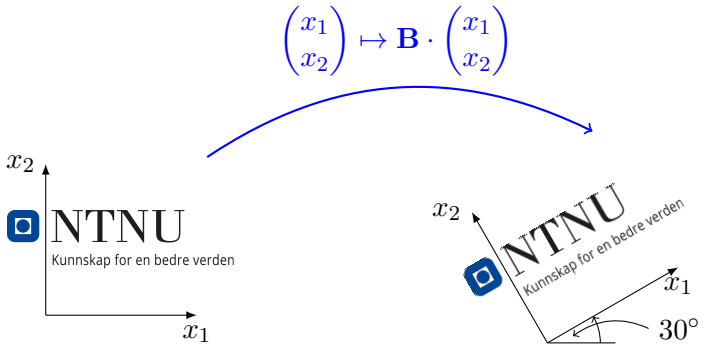
Find the values $\mathbf{A} \cdot \boldsymbol{e}_1$, $\mathbf{A} \cdot \boldsymbol{e}_2$, when $\boldsymbol{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are the two unit vectors in $\mathbb{R}^2$.

(b) What does the function $f\colon \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \mathbf{A} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ (dvs. $f(x_1, x_2) = \mathbf{A} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$) do *geometrically* to a vector $\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$?

(c) Find a matrix $\mathbf{B} \in \mathbb{R}^{2\times 2}$ such that

$$g\colon \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \mathbf{B} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

corresponds to a rotation of $\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ about the angle 30°.



Hint: • Consider part (a): The matrix must image the unit vector to their images.

• The following holds $\cos 45° = \sin 45°$. Search for "rotation matrix".

Deadline: Sunday, February 13th (digitally as one pdf-file via Blackboard)