Eksamen 2016 H

Oppg. 4 a)

$$f(x,y) = x - y \qquad D_f = \{(x,y) : x^2 + y^2 \leq 1\}$$

Finn globale max- og min-pkt for f på D_f.

$$\frac{\partial f}{\partial x} = 1$$

$$\nabla f(x,y) \neq \bar{0} \text{ for alle } (x,y).$$

$$\frac{\partial f}{\partial y} = -1$$

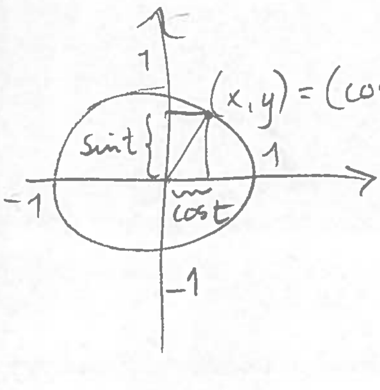
$\supset$: ingen kritiske punkter i det indre av def.m.

f har ingen singulære punkter.

<u>Randa</u>:

Parametriserer sirkelen:

$$x = \cos t \qquad t \in [0, 2\pi]$$



$$y = \sin t$$

$$\dot{g}(t) = \cos t - \sin t$$

$$g'(t) = -\sin t - \cos t$$

$$g'(t) = 0 \iff -\sin t - \cos t = 0$$

$$\Updownarrow$$

$$\sin t + \cos t = 0$$

$$\Updownarrow \text{ div. med } \cos t$$

$$v = \frac{\text{grader}}{180}\pi$$

$$\tan t + 1 = 0$$

$$\Updownarrow$$

$$\tan t = -1$$

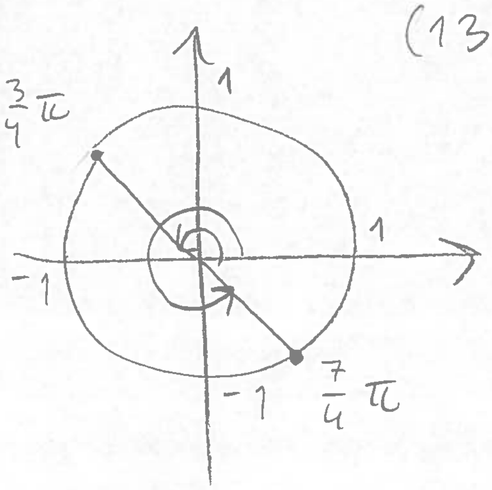
$$\Updownarrow$$

$$t = \frac{3}{4}\pi + k\pi$$

): $t \in [0, 2\pi]$ vil vi ha

$$t = \frac{3}{4}\pi \quad \text{og} \quad t = \frac{7}{4}\pi \quad (k=0 \text{ og } k=1)$$

$(135°)$ $(315°)$



$\therefore\; t = \frac{3}{4}\pi \Rightarrow x = \cos\frac{3}{4}\pi = -\frac{\sqrt{2}}{2}$

$$y = \sin\frac{3}{4}\pi = \frac{\sqrt{2}}{2}$$

$t = \frac{7}{4}\pi \Rightarrow x = \cos\frac{7}{4}\pi = \frac{\sqrt{2}}{2}$

$$y = \sin\frac{7}{4}\pi = -\frac{\sqrt{2}}{2}$$

$\therefore$ To pkt:

$$f\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = \underline{-\sqrt{2}} \to \text{Globalt } \underline{\text{min}}$$

$$f\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \underline{\sqrt{2}} \to \text{Globalt } \underline{\text{max.}}$$

b) Beregn den retningsderiverte $D_{\bar{u}}f$ i pkt $\left(\frac{1}{2}, \frac{1}{2}\right)$ i retningen av $\bar{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

$$D_{\bar{u}} f(a,b) = \nabla f(a,b) \cdot \frac{\bar{u}}{|\bar{u}|}$$

$$\bar{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad |\bar{u}| = \sqrt{1^2 + 2^2} = \underline{\sqrt{5}}$$

$$\frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$D_{\bar{u}} f\left(\frac{1}{2}, \frac{1}{2}\right) = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \cdot \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{\sqrt{5}}(1-2)$$

$$= \underline{-\frac{1}{\sqrt{5}}}$$

Får samme retn. derivert for alle $(x,y) \in D_f$ fordi grafen er et <u>plan</u>.

Øker mest i retning av gradienten

$\nabla f = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (uavh. av (x,y) fordi grafen er et plan).