Oppgaver fra øving 5, 2011V

$\boxed{9.2.46}$ $A = \begin{bmatrix} -1 & 3 \\ 0 & 3 \end{bmatrix}$ $\det A = \begin{vmatrix} -1 & 3 \\ 0 & 3 \end{vmatrix}$

$= -1 \cdot 3 - 3 \cdot 0 = -3$

$\Rightarrow$ A er inverterbar siden $\det A = -3 \neq 0$.

$\boxed{9.2.50}$ $A = \begin{bmatrix} a & 8 \\ 2 & 4 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

Husk! • $AX = B$ har entydig Løsning $\Leftrightarrow$ $\det A \neq 0$

• $AX = B$ har ingen eller uendlig mange Løsninger $\Leftrightarrow$ $\det A = 0$.

(a) $\det A = a \cdot 4 - 2 \cdot 8 = 4a - 16$

$\det A \neq 0 \Leftrightarrow a \neq 4$. Derfor har $AX = B$ entydig Løsning $\Leftrightarrow a \neq 4$.

(b) $a = 4$,

$\det A = \begin{vmatrix} 4 & 8 \\ 2 & 4 \end{vmatrix} = 4 \cdot 4 - 8 \cdot 2 = 0 \Rightarrow$ $AX = B$ har ingen | ∞ løsning.

$AX = B$, $\begin{bmatrix} 4 & 8 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, $\begin{bmatrix} 4x_1 + 8x_2 \\ 2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$\Rightarrow$ $4x_1 + 8x_2 = b_1$
$2x_1 + 4x_2 = b_2 \quad \cdot -2$

$$\left.\begin{aligned} 4x_1 + 8x_2 &= b_1 \\ -4x_1 - 8x_2 &= -2b_2 \end{aligned}\right\} +$$

$0 = b_1 - 2b_2$

(i) ∞ løsning hvis $b_1 = 2b_2$

(ii) ingen løsning hvis $b_1 \neq 2b_2$.

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$\boxed{9.2.54}$ $\quad D = \begin{bmatrix} -3 & 6 \\ -4 & 8 \end{bmatrix}$ $\quad \det D = -3 \cdot 8 - 6 \cdot (-4) = 0$

$\Rightarrow$ D er ikke inverterbar siden $\det D = 0$.

Husk! $DX = \underline{0}$ har ikke trivial løsning $(x \neq \underline{0})$

$\iff \det A = 0$

La $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. $\quad DX = \underline{0}$, $\quad \begin{bmatrix} -3 & 6 \\ -4 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \left.\begin{aligned} -3x_1 + 6x_2 &= 0 \\ -4x_1 + 8x_2 &= 0 \end{aligned}\right\}$ $\quad x_1 = 2x_2$, $\quad$ sett $x_2 = t \in \mathbb{R}$,

$x_1 = 2t$

$\Rightarrow \quad x = (x_1, x_2) = (t, 2t)$, $t \in \mathbb{R}$, er løsningene.

$\Rightarrow \quad DX = \underline{0}$ har uendlig mange løsning

$\boxed{9.2.57}$ $\quad A = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -2 & 0 \\ -1 & 1 & 2 \end{bmatrix}$

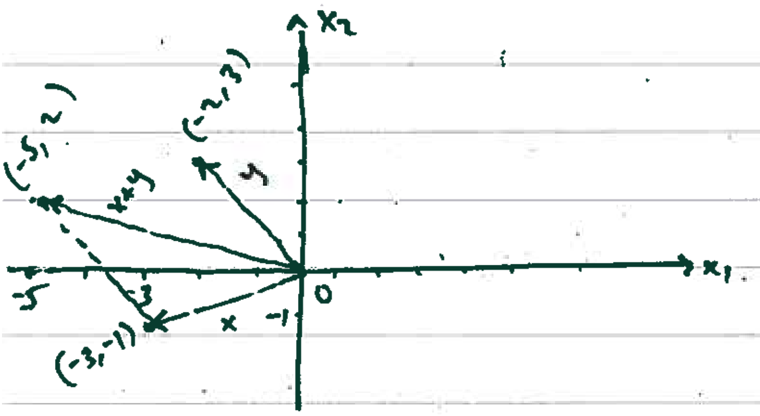
$$\left[\begin{array}{ccc|ccc} -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \cdot -1 \qquad \left[\begin{array}{ccc|ccc} -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & -1 & 0 & +1 \end{array}\right] \cdot 1/2$$

$$\left[\begin{array}{ccc|ccc} -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & -1 & 1/2 & 1 \end{array}\right] \cdot 1/3 \qquad \left[\begin{array}{ccc|ccc} -1 & 0 & 0 & 2/3 & 1/6 & 1/3 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & -1 & 1/2 & 1 \end{array}\right] \begin{array}{l} \cdot -1 \\ \cdot -1/2 \\ \cdot 1/3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2/3 & -1/6 & -1/3 \\ 0 & 1 & 0 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/3 & +1/6 & 1/3 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} -2/3 & -1/6 & -1/3 \\ 0 & -1/2 & 0 \\ -1/3 & +1/6 & 1/3 \end{bmatrix}$$

Oppgaver fra øving 5, 2011V

$\boxed{9.3.22}$ $\quad x = \begin{bmatrix} -3 \\ -1 \end{bmatrix} \quad y = \begin{bmatrix} -2 \\ 3 \end{bmatrix} \quad x+y = \begin{bmatrix} -3-2 \\ -1+3 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$



$\boxed{9.3.42}$ $\quad R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad R_{\pi/3} = \begin{bmatrix} \cos \pi/3 & -\sin \pi/3 \\ \sin \pi/3 & \cos \pi/3 \end{bmatrix}$

$x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \qquad = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$

$R_{\pi/3}\, x = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 + \sqrt{3}/2 \\ 2\sqrt{3} - 1/2 \end{bmatrix}$