oppgaver fra øving 4, 2011v

$\boxed{4.2.7}$

$$2A+3B-C = 2\begin{bmatrix}1 & 3 & -1\\ 2 & 4 & 1\\ 0 & -2 & 2\end{bmatrix} + 3\begin{bmatrix}5 & -1 & 4\\ 2 & 0 & 1\\ 1 & -3 & -3\end{bmatrix} - \begin{bmatrix}-2 & 0 & 4\\ 1 & -3 & 1\\ 0 & 0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}2 & 6 & -2\\ 4 & 8 & 2\\ 0 & -4 & 4\end{bmatrix} + \begin{bmatrix}15 & -3 & 12\\ 6 & 0 & 3\\ 3 & -9 & -9\end{bmatrix} - \begin{bmatrix}-2 & 0 & 4\\ 1 & -3 & 1\\ 0 & 0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}2+15-(-2) & 6-3-0 & -2+12-4\\ 4+6-1 & 8+0-(-3) & 2+3-1\\ 0+3-0 & -4-9-0 & 4-9-2\end{bmatrix}$$

$$= \begin{bmatrix}19 & 3 & 6\\ 9 & 11 & 4\\ 3 & -13 & -7\end{bmatrix} \quad //$$

$\boxed{4.2.25}$ $\quad A = \begin{bmatrix}1 & 3\\ 0 & -2\end{bmatrix}$ $\quad 2\times 2$ - matrise

$B = \begin{bmatrix}1 & 2 & 0 & -1\\ 2 & 1 & 3 & 0\end{bmatrix}$ $\quad 2\times 4$ - matrise

B' $\qquad B' = \begin{bmatrix}1 & 2\\ 2 & 1\\ 0 & 3\\ -1 & 0\end{bmatrix}$

$$AB = \underbrace{\begin{bmatrix}1 & 3\\ 0 & -2\end{bmatrix}}_{2\times 2}\underbrace{\begin{bmatrix}1 & 2 & 0 & -1\\ 2 & 1 & 3 & 0\end{bmatrix}}_{2\times 4} = \begin{bmatrix}1\cdot1+3\cdot2 & 1\cdot2+3\cdot1 & 1\cdot0+3\cdot3 & 1\cdot-1+3\cdot0\\ 0\cdot1-2\cdot2 & 0\cdot2-2\cdot1 & 0\cdot0-2\cdot3 & 0\cdot-1-2\cdot0\end{bmatrix}$$

$$= \begin{bmatrix}7 & 5 & 9 & -1\\ -4 & -2 & -6 & 0\end{bmatrix} \quad //$$

$$B'A = \underbrace{\begin{bmatrix}1 & 2\\ 2 & 1\\ 0 & 3\\ -1 & 0\end{bmatrix}}_{4\times 2}\underbrace{\begin{bmatrix}1 & 3\\ 0 & -2\end{bmatrix}}_{2\times 2} = \underbrace{\begin{bmatrix}1 & -1\\ 2 & 4\\ 0 & -6\\ -1 & -3\end{bmatrix}}_{4\times 2} \quad //$$

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$\boxed{9.2.30}$

$$AI_3 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & -1 & 2 \\ -1 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\cdot1+3\cdot0+0\cdot0 & 1\cdot0+3\cdot1+0\cdot0 & 1\cdot0+3\cdot0+0\cdot0 \\ 0\cdot1-1\cdot0+2\cdot0 & 0\cdot0-1\cdot1+2\cdot0 & 0\cdot0-1\cdot0+2\cdot1 \\ -1\cdot1-2\cdot0+1\cdot0 & -1\cdot0-2\cdot1+1\cdot0 & -1\cdot0-2\cdot0+1\cdot1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 0 \\ 0 & -1 & 2 \\ -1 & -2 & 1 \end{bmatrix} = A //$$

på samme måte, $I_3 A = A$.

$\boxed{9.2.36}$

$$BA = \begin{bmatrix} -6/5 & 2/5 & 7/5 \\ 3/5 & -1/5 & -1/5 \\ 8/5 & -1/5 & -11/5 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 5 & 2 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -12/5+10/5+7/5 & -18/5+4/5+14/5 & -6/5+6/5+7/5\cdot0 \\ 6/5-5/5-1/5 & 9/5-2/5-2/5 & 3/5-3/5-1/5\cdot0 \\ 16/5-5/5-11/5 & 24/5-2/5-22/5 & 8/5-3/5-11/5\cdot0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Siden $BA = I_3$, er B inversen til A.

oppgaver fra øving 4, 2011v

$\boxed{9.2.37}$ $A = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$

Vi søker $B = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}$ slik at $AB = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$$\begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 & -y_1 + y_2 \\ 2x_1 + 3x_2 & 2y_1 + 3y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Leftrightarrow$ ① $\begin{cases} -x_1 + x_2 = 1 \\ 2x_1 + 3x_2 = 0 \end{cases}$ ② $\begin{cases} -y_1 + y_2 = 0 \\ 2y_1 + 3y_2 = 1 \end{cases}$

Løs for x_1 og x_2 fra ①:

$\begin{cases} -x_1 + x_2 = 1 & \cdot 2 \\ 2x_1 + 3x_2 = 0 \end{cases}$

$+ \begin{cases} -2x_1 + 2x_2 = 2 \\ 2x_1 + 3x_2 = 0 \end{cases}$

$5x_2 = 2$

$\Rightarrow x_2 = 2/5$

Fra $2x_1 + 3x_2 = 0$, $2x_1 + 3 \cdot 2/5 = 0$

$\Rightarrow x_1 = -3/5$

Løs for y_1 og y_2 fra ②

Fra $-y_1 + y_2 = 0$, $y_1 = y_2$

$\Rightarrow 2y_1 + 3(y_1) = 1$

$\Rightarrow 5y_1 = 1 \Rightarrow y_1 = 1/5$

og derfor $y_2 = 1/5$

Altså $A^{-1} = B = \begin{bmatrix} -3/5 & 1/5 \\ 2/5 & 1/5 \end{bmatrix}$ er inversen til A.