Oppgaver fra Øving 2, 2011V

8.1.7

$$\frac{ds}{dt} = \sqrt{3t+1}, \quad s(0) = 1$$

$$\int ds = \int \sqrt{3t+1}\, dt, \quad \text{men} \quad \int \sqrt{3t+1}\, dt \underset{du = 3dt}{\overset{u=3t+1}{=}} \frac{1}{3}\int \sqrt{u}\, du$$

$$= \frac{1}{3}\cdot\frac{2}{3} u^{3/2} + c$$

$$= \frac{2}{9}(3t+1)^{3/2} + c$$

$$s(t) = \frac{2}{9}(3t+1)^{3/2} + c$$

$$1 = s(0) = \frac{2}{9}(3\cdot 0 + 1)^{3/2} + c$$

$$\Rightarrow \quad c = 1 - 2/9 = 7/9$$

$$\Rightarrow \boxed{s(t) = \frac{2}{9}(3t+1)^{3/2} + \frac{7}{9}}$$

8.1.17

$$\frac{dN}{dt} = 0{,}3\,N(t), \quad N(0) = 20$$

$$\int \frac{dN}{N} = \int 0{,}3\, dt, \quad \ln|N| = 0{,}3t + c_1$$

$$|N| = e^{c_1} e^{0{,}3t}$$

$$\Rightarrow \quad N(t) = C e^{0{,}3t} \quad \text{der} \quad C = \pm e^{c_1}$$

$$20 = N(0) = C \cdot e^{0{,}3\cdot 0} = C \quad \Rightarrow \quad C = 20$$

$$\Rightarrow \boxed{N(t) = 20\, e^{0{,}3t}}$$

- For $t = 5$: $N(5) = 20\, e^{0{,}3\cdot 5} = \underline{\underline{20\, e^{1{,}5}}} \approx \underline{\underline{90}}$

Oppgaver fra øving 2, 2011V

$\boxed{8.1.30}$ $\frac{dy}{dt} = \frac{1}{2}y^2 - 2y$; $y_0 = -3$ når $t_0 = 0$

$$\int \frac{dy}{y^2-4y} = \frac{1}{2}\int dt \quad \text{fordi } \tfrac{1}{2}y^2 - 2y = \tfrac{1}{2}(y^2-4y)$$

$$\frac{1}{y^2-4y} = \frac{1}{y(y-4)} = \frac{A}{y} + \frac{B}{y-4} = \frac{Ay-4A+By}{y(y-4)} \quad \text{for alle } y$$

$$\Rightarrow \quad 1 = Ay - 4A + By = (A+B)y - 4A \quad \text{for alle } y$$

$$\Rightarrow \quad -4A = 1 \quad \Rightarrow \quad A = -1/4$$
$$A + B = 0 \quad \Rightarrow \quad B = -A = 1/4$$

Altså $\frac{1}{y^2-4y} = \frac{-1/4}{y} + \frac{1/4}{y-4}$

og $\int \frac{dy}{y^2-4y} = \int\left(\frac{-1/4}{y} + \frac{1/4}{y-4}\right)dy = -\frac{1}{4}\ln|y| + \frac{1}{4}\ln|y-4|$
$= \frac{1}{4}\ln\left|\frac{y-4}{y}\right|$

Derfor

$$\frac{1}{4}\ln\left|\frac{y-4}{y}\right| = \frac{1}{2}t + C_1$$

$$\ln\left|\frac{y-4}{y}\right| = 2t + 2C_1$$

$$\Rightarrow \quad \frac{y-4}{y} = C e^{2t} \quad \text{der} \quad C = \pm e^{2C_1}$$

$$\Rightarrow \quad y(t) = \frac{4}{1-Ce^{2t}}$$

- $y_0 = -3$ når $t_0 = 0$ betyr at $y(0) = -3$

$$\Rightarrow \quad -3 = \frac{4}{1-Ce^{2\cdot 0}} = \frac{4}{1-C} \quad \Rightarrow -3 + 3C = 4 \Rightarrow C = 7/3$$

Altså

$$\boxed{y(t) = \frac{4}{1-\frac{7}{3}e^{2t}}}$$ er løsningen!

Oppgaver fra Øving 2, 2011v

$\boxed{8.1.49}$

$$\frac{dy}{dx} = \frac{y+1}{x-1}, \quad y_0 = 5 \text{ når } x_0 = 2$$

$$\int \frac{dy}{y+1} = \int \frac{dx}{x-1}. \qquad \ln|y+1| = \ln|x-1| + C_1$$

$$y+1 = C(x-1), \quad C = \pm e^{C_1}$$

$$\Rightarrow \quad y(x) = C(x-1) - 1$$

- Men $y_0 = 5$ når $x_0 = 2 \Leftrightarrow y(2) = 5$

Altså

$$5 = y(2) = C(2-1) - 1 \Rightarrow \underline{C = 6}$$

og $y(x) = 6(x-1) - 1$, $\boxed{y(x) = 6x - 7}$

Kontroll: v.s. $\frac{dy}{dx} = 6$ h.s. $\frac{y+1}{x-1} = \frac{6x-7+1}{x-1} = \frac{6(x-1)}{x-1} = 6$

v.s. = h.s. ⇒ OK!

også $y(2) = 6 \cdot 2 - 7 = 12 - 7 = 5 \Rightarrow$ OK!

Oppgaver fra øving 2, 2011V

$\boxed{\text{2006V, oppgave 1}}$

$$3x^2 \frac{dx}{dt} + t = 0, \qquad x = 2 \text{ når } t = 4$$
$$\Leftrightarrow \quad x(4) = 2$$

Omskrive: $3x^2\,dx = -t\,dt$

$$\int 3x^2\,dx = \int -t\,dt, \qquad x^3(t) = -\frac{t^2}{2} + C$$

- $x(4) = 2 \Rightarrow 8 = x^3(4) = -\frac{4^2}{2} + C = -8 + C$

$$\Rightarrow C = 16$$

Altså $x^3(t) = -\frac{t^2}{2} + 16$

$$\Rightarrow \boxed{x(t) = \left(-\frac{t^2}{2} + 16\right)^{1/3}}$$

er løsningen

Derfor: $x(0) = 16^{1/3} \approx 2{,}52$

Kontroll:

$$\frac{dx}{dt} = \frac{1}{3}\left(\underbrace{-\frac{t^2}{2} + 16}_{x^3}\right)^{-\frac{2}{3}} \cdot -t$$
$$= \frac{1}{3} x^{-2} \cdot -t$$
$$\Rightarrow 3x^2 \frac{dx}{dt} = -t$$
$$\Rightarrow 3x^2 \frac{dx}{dt} + t = 0 \Rightarrow \text{OK!}$$

også $x(4) = 2$.

$\boxed{\text{2006V, oppgave 2}}$

$$\frac{dy}{dt} = -2(1-y)(2-y)y$$

La $g(y) = -2(1-y)(2-y)y$. $g(y) = 0 \Leftrightarrow y = 0$ eller $y = 1$ eller $y = 2$

$\Rightarrow$ Likevektspunktene er $y = 0$, $y = 1$ og $y = 2$.

Omskrive: $g(y) = -2y^3 + 6y^2 - 4y$. Da er $g'(y) = -6y^2 + 12y - 4$.

- $y = 0$: $g'(0) = -4 < 0 \Rightarrow y = 0$ er lokal stabilt.
- $y = 1$: $g'(1) = 2 > 0 \Rightarrow y = 1$ er ustabilt
- $y = 2$: $g'(2) = -4 < 0 \Rightarrow y = 2$ er lokal stabilt

Derfor: vi har to stabilt og ett ustabilt l.k.p.