

Oppgaver fra øving 1, 2011V

7.1.2 $\int 3x^2 \sqrt{x^3+1} dx$, $u = x^3 + 1$

$$\frac{du}{dx} = 3x^2$$

$$\Rightarrow du = 3x^2 dx$$

$$\int 3x^2 \sqrt{x^3+1} dx = \int \sqrt{x^3+1} \underbrace{3x^2 dx}_{du}$$

$$= \int \sqrt{u} du = \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{3} (x^3+1)^{3/2} + C$$

7.1.13 $\int \frac{x+2}{x^2+4x} dx$, $u = x^2 + 4x$

$$\frac{du}{dx} = 2x+4 \Rightarrow du = 2(x+2)dx$$

$$\Rightarrow \frac{du}{2} = (x+2)dx$$

$$\int \frac{x+2}{x^2+4x} dx = \int \frac{(x+2) dx}{x^2+4x} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln|u| + C$$

$$= \frac{1}{2} \ln|x^2+4x| + C$$

7.2.13 $\int x \ln(3x) dx$ sette $u = \ln(3x)$ $v' = x$

$$\Rightarrow u' = \frac{1}{x} \quad v = \frac{x^2}{2}$$

$$\int x \ln(3x) dx = \int \underbrace{\ln(3x)}_u \underbrace{x dx}_{v'}$$

Delvis integrasjon

$$= \underbrace{\ln(3x)}_u \cdot \underbrace{\frac{x^2}{2}}_v - \int \underbrace{\frac{1}{x}}_{u'} \cdot \underbrace{\frac{x^2}{2}}_v dx$$

$$= \frac{x^2}{2} \ln(3x) - \frac{1}{2} \int x dx$$

$$= \frac{x^2}{2} \ln(3x) - \frac{x^2}{4} + C$$

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$$\boxed{7.3.29} \quad \int \frac{1}{x^2-9} dx$$

merk at $x^2-9 = (x-3)(x+3)$

Da er

$$\frac{1}{x^2-9} = \frac{1}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

$$= \frac{A(x+3) + B(x-3)}{(x-3)(x+3)}$$

$$= \frac{(A+B)x + 3(A-B)}{(x-3)(x+3)} \quad \text{for alle } x$$

⇒ må ha

$$1 = (A+B)x + 3(A-B) \quad \text{for alle } x$$

$$\Rightarrow \begin{cases} 3(A-B) = 1 \\ A+B = 0 \end{cases} \Rightarrow \begin{cases} A-B = \frac{1}{3} \\ A+B = 0 \end{cases}$$

$$\frac{2A = \frac{1}{3}}{\quad} \Rightarrow \underline{A = \frac{1}{6}}$$

$$A+B=0 \Rightarrow B = -A \Rightarrow \underline{B = -\frac{1}{6}}$$

Attså

$$\frac{1}{x^2-9} = \frac{1/6}{x-3} + \frac{-1/6}{x+3}$$

$$\int \frac{1}{x^2-9} dx = \int \left(\frac{1/6}{x-3} + \frac{-1/6}{x+3} \right) dx$$

$$= \frac{1}{6} \left[\int \frac{1}{x-3} dx - \int \frac{1}{x+3} dx \right]$$

$$= \frac{1}{6} (\ln|x-3| - \ln|x+3|) + C$$

$$= \underline{\underline{\frac{1}{6} \ln \left| \frac{x-3}{x+3} \right|}} + C. \quad \text{N.B. } \ln a - \ln b = \ln a/b$$