



Most of the assignments so far had 4 questions. In this question sheet you can find 9 questions. **You need 3 questions answered correctly** in order to have the assignment approved.

You can also deal with the remaining ones for practice/revision. (Oppgave 9 is a little too hard, don't spend too much time on it.)

1 Evaluate the following integrals.

a) $\int_0^{\pi/2} e^{-x} \sin(2x) dx$

b) $\int_0^{\ln 2} \frac{e^x - e^{-x}}{e^x + e^{-x}} dx.$

2 Let

$$f(x) = \int_{\sqrt{x}}^{x^2} \left(u^2 - \frac{1}{\sqrt{u}} \right) du, \quad x > 0.$$

Find $f'(x)$.

3 Find the limit

$$\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}.$$

4 Prove that the equation

$$(x^2 - 4)e^x = (x^2 + 1) \ln \left(\frac{1}{1+x} \right)$$

has at least one solution in the interval $(0, 2)$.

5 Let

$$g : \mathbb{R} \rightarrow \mathbb{R}, \quad g(x) = (x + 1)e^{-x}.$$

Find the domain of g , the intervals where g is monotonic, the intervals where it is convex/concave and its asymptotes. Sketch the graph of g .

6 Find the area of the domain in the two-dimensional plane that is surrounded by the graph of the function $f(x) = 1 - x^2$, the tangent line of the graph at the point $(0, f(0))$ and the tangent line of the graph at the point $(1, f(1))$.

7 Find the limit

$$\lim_{x \rightarrow 1} \frac{1}{x - 1} \int_x^{x^2} \frac{\sin u}{u} du.$$

8 Find $a \in \mathbb{R}$ such that

$$\int_a^{\infty} e^{-2x} dx = \frac{1}{4}.$$

9 (*) Which of the following numbers is bigger: e^π or π^e ?