1 Regn ut integralene

a) $\displaystyle\int_{-1}^{1} \frac{e^x - e^{-x}}{2}\,\mathrm{d}x.$

b) $\displaystyle\int_{1}^{e} \frac{\ln x}{x}\,\mathrm{d}x.$

c) $\displaystyle\int_{0}^{1} 2x^2 e^{x^3+1}\,\mathrm{d}x.$

d) $\displaystyle\int_{0}^{\frac{\pi}{2}} x^2 \cos x\,\mathrm{d}x.$

e) $\displaystyle\int_{-2}^{-1} \frac{1}{3x+2}\,\mathrm{d}x.$

f) $\displaystyle\int_{1}^{e} x \ln x\,\mathrm{d}x.$

Løsning.
a)

$$\begin{aligned}\int_{-1}^{1} \frac{e^x - e^{-x}}{2}\,\mathrm{d}x &= \frac{1}{2}\int_{-1}^{1} e^x\,\mathrm{d}x + \frac{1}{2}\int_{-1}^{1} (-e^{-x})\,\mathrm{d}x \\ &= \frac{1}{2}[e^x]_{-1}^{1} + \frac{1}{2}[e^{-x}]_{-1}^{1} \\ &= 0.\end{aligned}$$

b) We set $u = \ln x \Rightarrow \mathrm{d}u = \frac{1}{x}\,\mathrm{d}x.$
$x_1 = 1 \Rightarrow u_1 = 0,\ x_2 = e \Rightarrow u_2 = 1.$

$$\int_{1}^{e} \frac{\ln x}{x}\,\mathrm{d}x = \int_{0}^{1} u\,\mathrm{d}u = \left[\frac{u^2}{2}\right]_0^1 = \frac{1}{2}.$$

c) We set $u = x^3 + 1 \Rightarrow \mathrm{d}u = 3x^2\,\mathrm{d}x \Rightarrow 2x^2\,\mathrm{d}x = \frac{2}{3}\,\mathrm{d}u.$
$x_1 = 0 \Rightarrow u_1 = 1, \quad x_2 = 1 \Rightarrow u_2 = 2.$

$$\int_{0}^{1} 2x^2 e^{x^3+1}\,\mathrm{d}x = \int_{1}^{2} \frac{2}{3} e^u\,\mathrm{d}u = \left[\frac{2}{3}e^u\right]_1^2 = \frac{2}{3}(e^2 - e).$$

d) We have

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} x^2 \cos x \,\mathrm{d}x &= \int_0^{\frac{\pi}{2}} x^2 (\sin x)' \,\mathrm{d}x \\
&= \Big[x^2 \sin x\Big]_0^{\pi/2} - \int_0^{\pi/2} (x^2)'(\sin x) \,\mathrm{d}x \\
&= \frac{\pi^2}{4} - \int_0^{\pi/2} 2x \sin x \,\mathrm{d}x \\
&= \frac{\pi^2}{4} - \int_0^{\pi/2} 2x(-\cos x)' \,\mathrm{d}x \\
&= \frac{\pi^2}{4} - [-2x\cos x]_0^{\pi/2} + \int_0^{\pi/2} (2x)'(-\cos x) \,\mathrm{d}x \\
&= \frac{\pi^2}{4} - 2\int_0^{\pi/2} \cos x \,\mathrm{d}x \\
&= \frac{\pi^2}{4} - 2[\sin x]_0^{\pi/2} \\
&= \frac{\pi^2}{4} - 2.
\end{aligned}$$

e) We set $u = 3x + 2 \Rightarrow \mathrm{d}u = 3\,\mathrm{d}x$.
Then $x = -2 \Rightarrow u = -4$ and $x = -1 \Rightarrow u = -1$.

$$\int_{-2}^{-1} \frac{1}{3x+2} \,\mathrm{d}x = \int_{-4}^{-1} \frac{1}{3u} \,\mathrm{d}u = \frac{1}{3}[\ln|u|]_{-4}^{-1} = \frac{1}{3}(\ln 1 - \ln 4) = -\frac{\ln 4}{3}\cdot$$

f) We have

$$\begin{aligned}
\int_1^e x \ln x \,\mathrm{d}x &= \int_1^e \left(\frac{x^2}{2}\right)' \ln x \,\mathrm{d}x \\
&= \left[\frac{x^2}{2} \ln x\right]_1^e - \int_1^e \frac{x^2}{2} (\ln x)' \,\mathrm{d}x \\
&= \frac{e^2}{2} - \int_1^e \frac{x}{2} \,\mathrm{d}x \\
&= \frac{e^2}{2} - \left[\frac{x^2}{4}\right]_1^e \\
&= \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} \\
&= \frac{e^2+1}{4}\cdot
\end{aligned}$$

2 Forklar hvorfor integralet

$$\int_{-3}^{3} \frac{1}{x^4} dx$$

ikke bør løses ved å finne den antideriverte og sette inn grensene 3 og -3.

Løsning.
The function

$$f(x) = \frac{1}{x^4}$$

is not continuous on the interval $[-4,4]$.