Eksamen MA0001h 2014

Oppg 1

$$2(e^x)^2 - e^{2x} = 2e^x - 1$$

$$\Leftrightarrow$$

$$e^{2x} - 2e^x + 1 = 0$$

$$\Leftrightarrow$$

$$0 = (e^x - 1)^2$$

$$\Leftrightarrow$$

$$e^x = 1 \quad \Leftrightarrow \quad \underline{\underline{x = 0}}$$

Oppg 2

$$\lim_{x\to\infty} \sqrt{4x^2+3x} - 2x$$

$$= \lim_{x\to\infty} \frac{(\sqrt{4x^2+3x} - 2x)(\sqrt{4x^2+3x} + 2x)}{\sqrt{4x^2+3x} + 2x}$$

$$= \lim_{x\to\infty} \frac{\cancel{4x^2} + 3x - \cancel{4x^2}}{\sqrt{4x^2+3x} + 2x}$$

$$= \lim_{x\to\infty} \frac{3}{\sqrt{4 + 3/x} + 2}, \qquad x > 0$$

$$= \frac{3}{\sqrt{4} + 2} = \underline{\underline{\frac{3}{4}}}$$

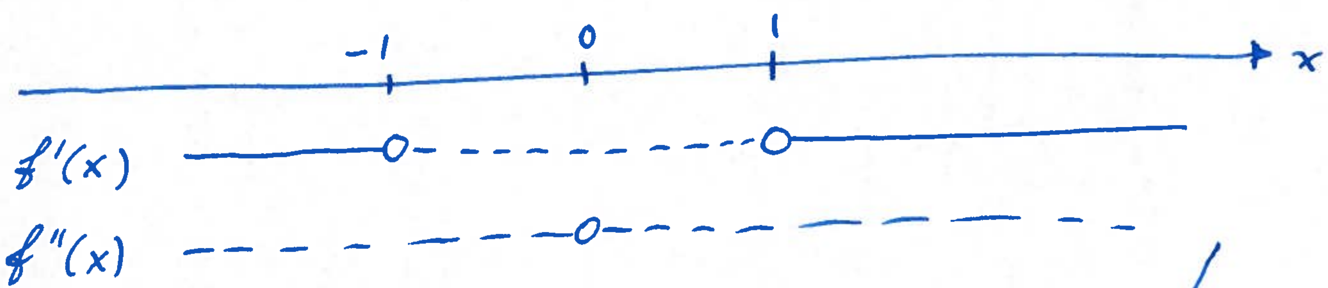
Oppg 3 La $g(x) = x^5 - 3x^3 - 1$

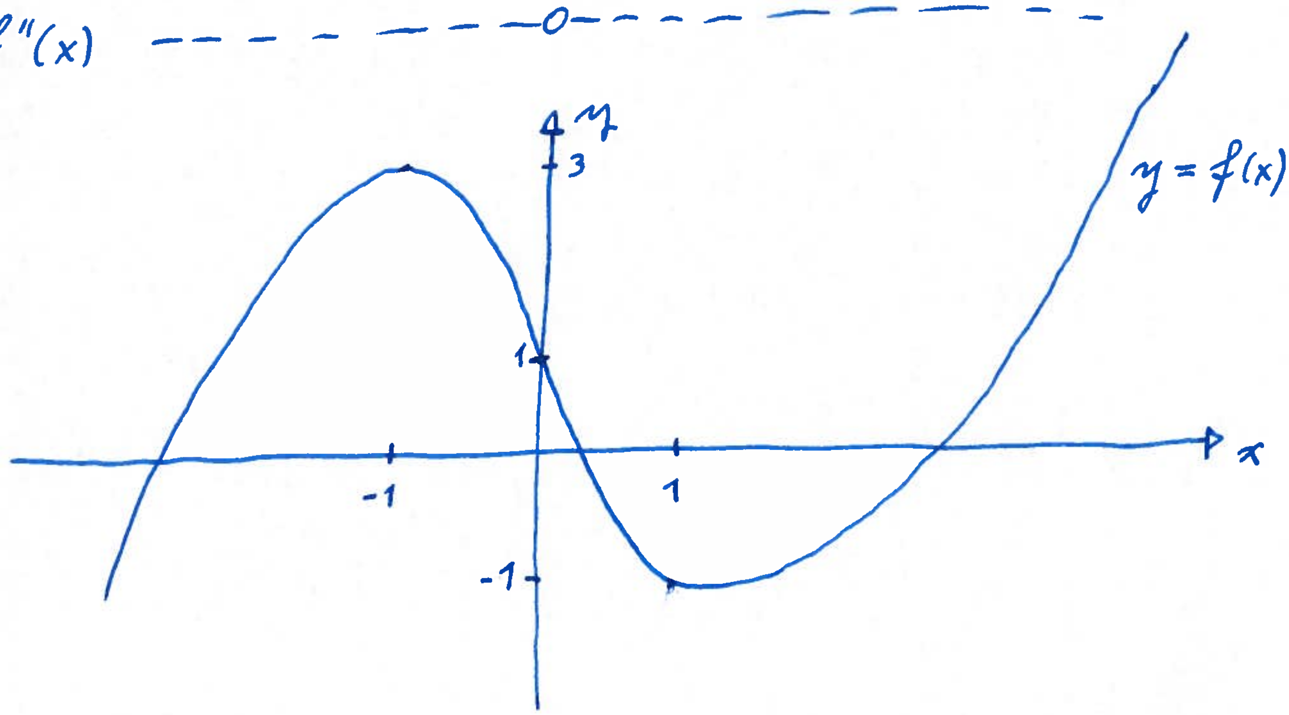
g er kontinuerlig og $\lim_{x \to -\infty} g(x) = -\infty$ og $\lim_{x \to \infty} g(x) = +\infty$. Ved IVT finner da en $c \in \mathbb{R}$ s.a. $g(c) = 0$. Dvs $c^5 - 3c^3 = 1$ $\blacksquare$

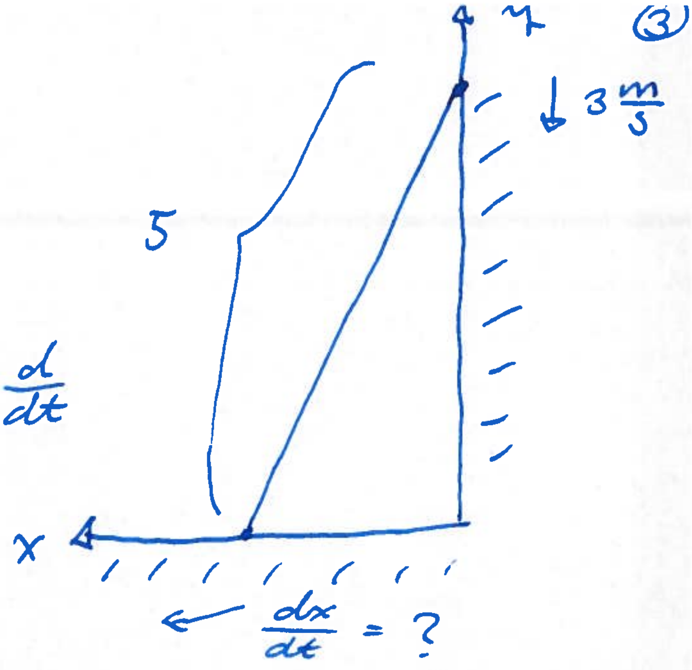
Oppg 4 $f(x) = x^3 - 3x + 1$

Vi har at

- $\lim_{x \to \pm\infty} f(x) = \pm\infty$
- $f'(x) = \overbrace{3x^2 - 3}^{3(x+1)(x-1)}$,): Kr. p. i $x = \pm 1$
- $f''(x) = 6x$,): Vendep. i $x = 0$
- $f(0) = 1$, $f(1) = -1$, $f(-1) = 3$







Oppg 5

- $x^2 + y^2 = 5^2$

$$\Rightarrow 0 = 2xx' + 2yy', \qquad ' = \frac{d}{dt}$$

$$\Rightarrow \frac{dx}{dt} = -\frac{y\frac{dy}{dt}}{x}$$

Når $y = 4$ er $x = 3$, så

$$\left.\frac{dx}{dt}\right|_{\substack{x=3\\ y=4\\ y'=-3}} = \frac{4\cdot 3}{3} = \underline{\underline{4\frac{m}{s}}}$$

Oppg 6 $\quad P'(t) = kP(t)$

$$\Rightarrow P(t) = P(0)e^{kt} = 1\cdot e^{kt}$$

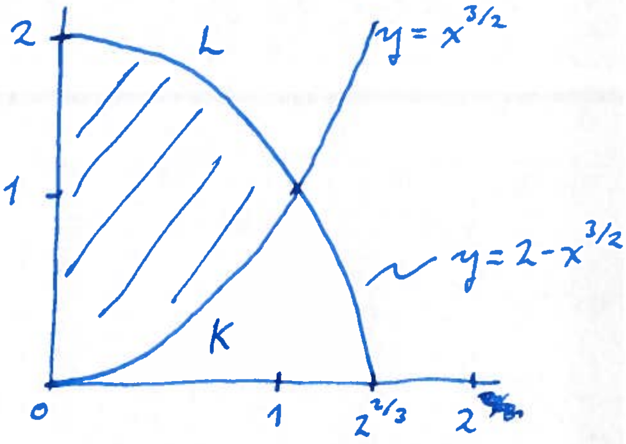
$$2 = P(1) = e^k$$

$$\Rightarrow P(2) = e^{2k} = (e^k)^2 = 2^2 = 4$$

Befolkningen er $\underline{\underline{4\,000\,000}}$ ved tiden $t = 2$

(4)

<u>Oppg 7</u> $y = x^{3/2}, \quad y = 2 - x^{3/2}$



a) $A = \int_0^1 2 - x^{3/2} - x^{3/2}\,dx$

$= \int_0^1 2 - 2x^{3/2}\,dx$

$= \Big/_0^1\, 2x - 2\frac{2}{5}x^{5/2} = 2 - \frac{4}{5}$

$= \underline{\underline{\frac{6}{5}}}$

b) $\frac{d}{dx}\left(\frac{8}{27}(1 + 9x/4)^{3/2}\right)$

$= \cancel{\frac{8}{27}}\cancel{\frac{3}{2}}(1 + 9x/4)^{1/2}\cancel{\frac{9}{4}} = \sqrt{1 + \frac{9}{4}x}$ $\qquad$ II

c) La K, L og M være lengdene av kurvene indikert i figuren

$K = \int_0^1 \sqrt{1 + (\frac{dy}{dx})^2}\,dx, \qquad \frac{dy}{dx} = \frac{3}{2}x^{1/2}, \; (\frac{dy}{dx})^2 = \frac{9}{4}x$

$= \int_0^1 \sqrt{1 + \frac{9}{4}x}\,dx$

$= \frac{8}{27}\Big/_0^1 (1 + \frac{9}{4}x)^{3/2}$ ved b)

$= \frac{8}{27}\left\{(1 + \frac{9}{4})^{3/2} - 1\right\} = \frac{8}{27}\left((\frac{13}{4})^{3/2} - 1\right) = \frac{8}{27}\left(\frac{13^{3/2}}{8} - 1\right)$

$= \frac{13^{3/2} - 8}{27} \overset{\text{CALC}}{\cong} \underline{1{,}44}$

Ved symmetri er $L = K$, $M = 2$, så omkretsen er

$$O = K + L + M$$
$$= 2K + 2 = 2(K+1)$$
$$= 2\left(\frac{13^{3/2} - 8 + 27}{27}\right) = \underline{\underline{2\,\frac{13^{3/2}+19}{27}}} \approx 4.88$$

<u>Oppg 8</u> $f(x) = -\ln(1-x)$, $f(0) = 0$

$$f'(x) = -\frac{1}{1-x}(-1) = \frac{1}{1-x}, \quad f'(0) = 1$$

$$f''(x) = -\frac{1}{(1-x)^2}(-1) = \frac{1}{(1-x)^2}, \quad f''(0) = 1$$

$$\Rightarrow \quad P_2(x) = f(0) + f'(0)(x-0) + \frac{1}{2}f''(0)(x-0)^2$$
$$= 0 + \underline{\underline{x + \frac{1}{2}x^2}}$$