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$$\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla \cdot \vec{F}) - \Delta \vec{F}$$

$$\nabla \times (\nabla \times \vec{F}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ \begin{vmatrix} \partial_y & \partial_z \\ F_2 & F_3 \end{vmatrix} & -\begin{vmatrix} \partial_x & \partial_z \\ F_1 & F_3 \end{vmatrix} & \begin{vmatrix} \partial_x & \partial_y \\ F_1 & F_2 \end{vmatrix} \end{vmatrix}$$

$$= \left(\partial_y \begin{vmatrix} \partial_x & \partial_y \\ F_1 & F_2 \end{vmatrix} + \partial_z \begin{vmatrix} \partial_x & \partial_z \\ F_1 & F_3 \end{vmatrix},\ \partial_z \begin{vmatrix} \partial_y & \partial_z \\ F_2 & F_3 \end{vmatrix} - \partial_x \begin{vmatrix} \partial_x & \partial_y \\ F_1 & F_2 \end{vmatrix},\right.$$

$$\left. - \partial_x \begin{vmatrix} \partial_x & \partial_z \\ F_1 & F_3 \end{vmatrix} - \partial_y \begin{vmatrix} \partial_y & \partial_z \\ F_2 & F_3 \end{vmatrix} \right)$$

$$= \left(\begin{vmatrix} 0 & -\partial_z & \partial_y \\ \partial_x & \partial_y & \partial_z \\ F_1 & F_2 & F_3 \end{vmatrix},\ \begin{vmatrix} \partial_z & 0 & -\partial_x \\ \partial_x & \partial_y & \partial_z \\ F_1 & F_2 & F_3 \end{vmatrix},\ \begin{vmatrix} -\partial_y & \partial_x & 0 \\ \partial_x & \partial_y & \partial_z \\ F_1 & F_2 & F_3 \end{vmatrix} \right)$$

$$= \left(-\partial_x \begin{vmatrix} -\partial_z & \partial_y \\ F_2 & F_3 \end{vmatrix} + \begin{vmatrix} -\partial_z & \partial_y \\ \partial_y & \partial_z \end{vmatrix} F_1,\ \partial_y \begin{vmatrix} \partial_z & -\partial_x \\ F_1 & F_3 \end{vmatrix} - \begin{vmatrix} \partial_z & -\partial_x \\ \partial_x & \partial_z \end{vmatrix} F_2,\right.$$

$$\left. -\partial_z \begin{vmatrix} -\partial_y & \partial_x \\ F_1 & F_2 \end{vmatrix} + \begin{vmatrix} -\partial_y & \partial_x \\ \partial_x & \partial_y \end{vmatrix} F_3 \right)$$

$$= \left(\partial_x(\partial_y F_2 + \partial_y F_3) - (\partial_y^2 + \partial_y^2) F_1,\ \partial_y(\partial_x F_1 + \partial_z F_3) - (\partial_x^2 + \partial_z^2) F_2,\right.$$

$$\left. \partial_z(\partial_x F_1 + \partial_y F_2) - (\partial_x^2 + \partial_y^2) F_3 \right)$$

$$= \left(\partial_x(\nabla \cdot \vec{F}) - \Delta F_1,\ \partial_y(\nabla \cdot \vec{F}) - \Delta F_2,\ \partial_z(\nabla \cdot \vec{F}) - \Delta F_3 \right)$$

$$= \nabla(\nabla \cdot \vec{F}) - \Delta \vec{F}$$

$$= \nabla(\nabla \cdot \vec{F}) - \Delta \vec{F}$$