(4)

$$I = \int_0^{2\pi}\int_0^{1-\cos(\theta)} r\cos(\theta)^2\, dr\, d\theta$$

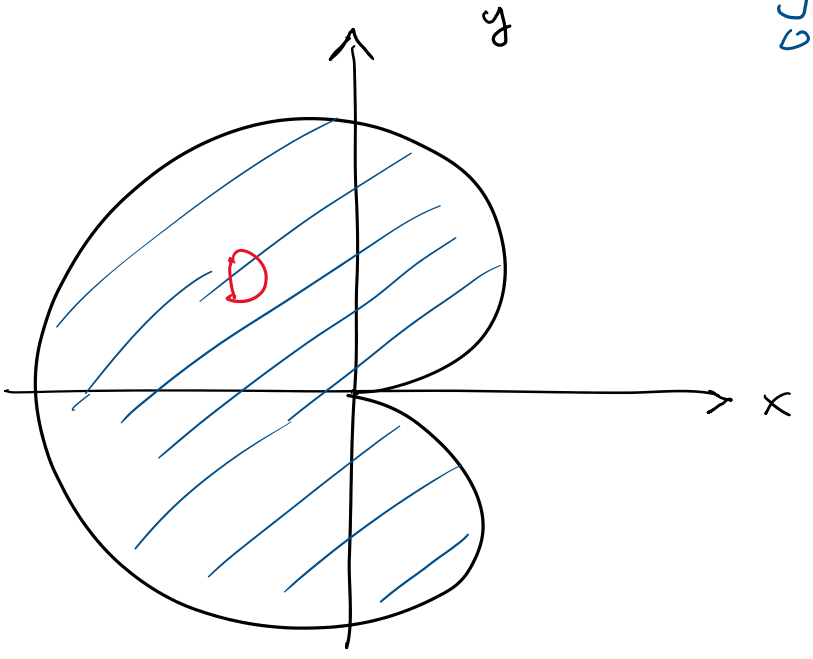
a) Beregn integralet

b) Skisser og angi D til kartesiske koordinater

$$I = \frac{1}{2}\int_0^{2\pi} (1-\cos(\theta))^2\cos(\theta)^2 d\theta = \frac{1}{2}\int_0^{2\pi}\left[\cos(\theta)^2 - 2\cos(\theta)^3 + \cos(\theta)^4\right] d\theta$$

$$= \frac{\pi}{2}\left[\frac{1}{2^{2-1}}\binom{2}{1} + \frac{1}{2^{4-1}}\binom{4}{2}\right] = \frac{7\pi}{8}$$

$$\int_0^{2\pi}\cos(\theta)^n d\theta = \begin{cases} 0 & n \text{ ulike} \\ \frac{1}{2^{n-1}}\binom{n}{n/2}\pi & n \text{ like} \end{cases}$$



(Kardioide)

$$r = 1-\cos(\theta)$$

$$r^2 = r - x$$

$$(x+r^2)^2 = r^2$$

$$(x+x^2+y^2)^2 = x^2+y^2$$

D er området innenfor kurven med likning

$r\,dr\,d\theta = dx\,dy$ og $\cos(\theta)^2 = \frac{r^2\cos(\theta)^2}{r^2} = \frac{x^2}{x^2+y^2}$

Så $I = \iint_D \frac{x^2}{x^2+y^2}\, dx\, dy$