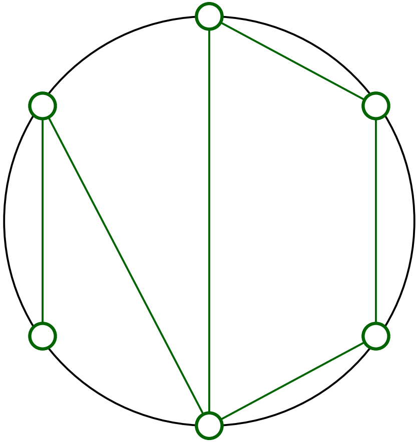
Flips of dissections and mutations of silting objects

Yann Palu

Université de Picardie Jules Verne

Fryday 10th January 2025 - Flash Talks NTNU

Accordions



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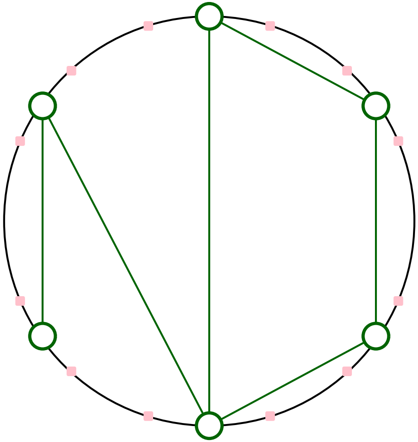

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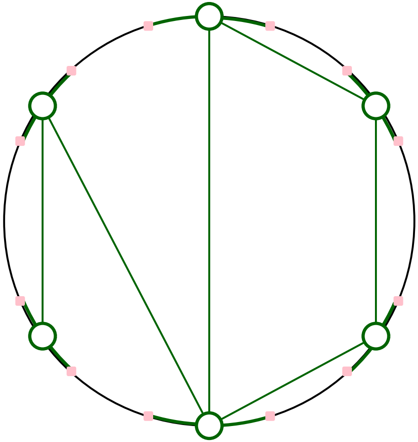

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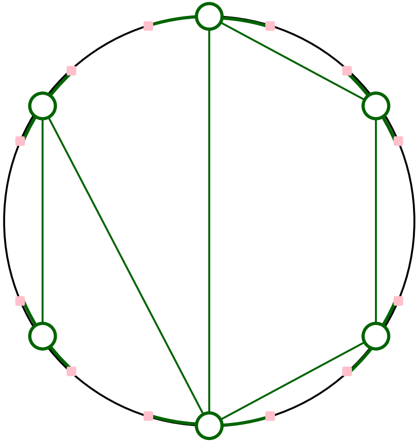

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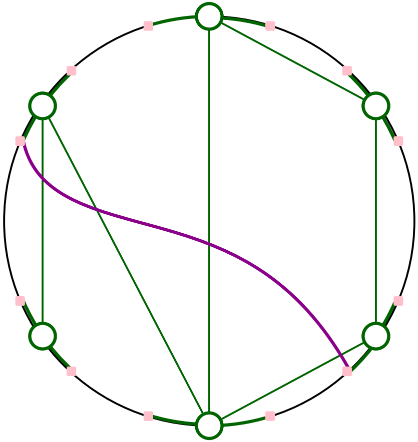

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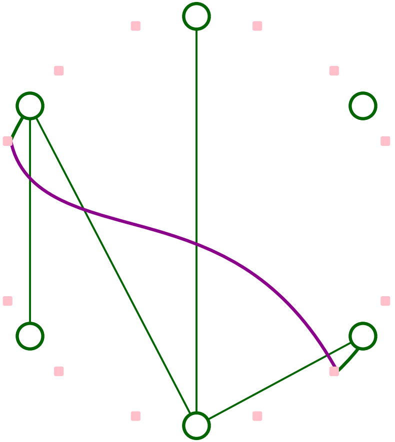

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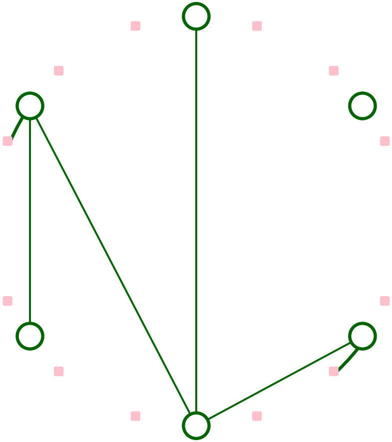

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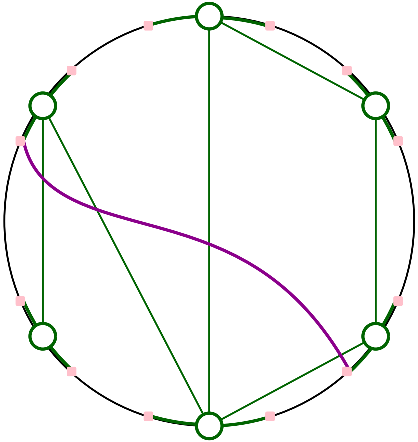

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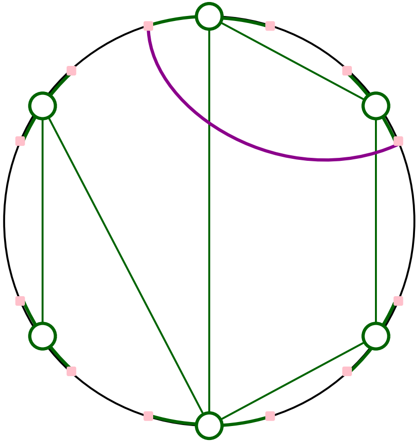

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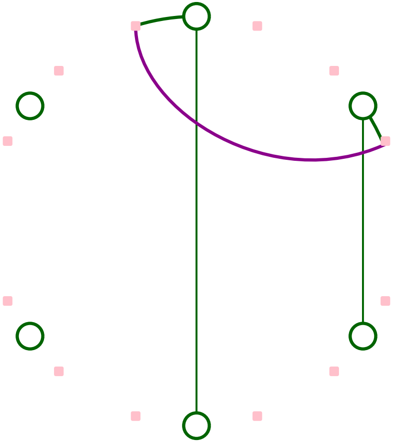

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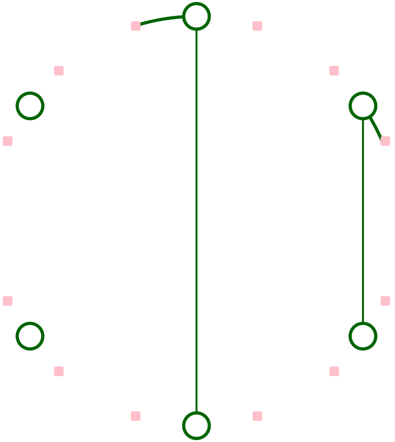

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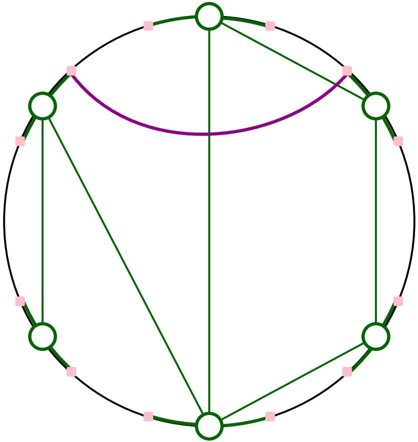

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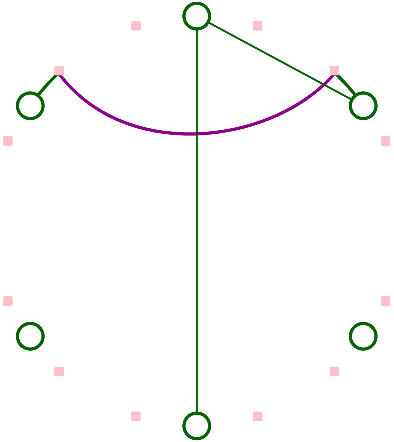

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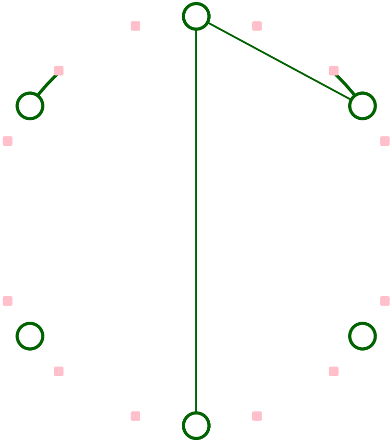

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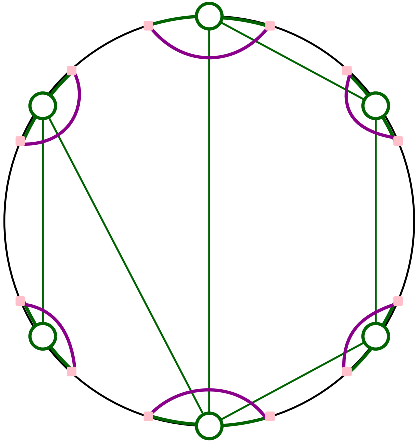

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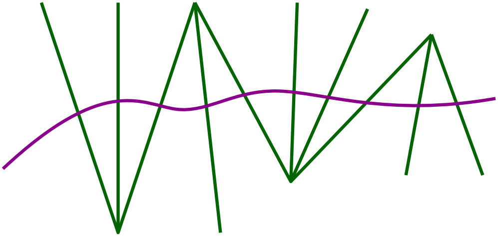

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Theorem (Garver–McConville; Manneville–Pilaud)

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2. *If $a \in \mathcal{T}$ is a relevant accortion, there exists a unique accordion $b \neq a$ such that $(\mathcal{T} \setminus \{a\}) \cup \{b\}$ is a maximal $\mathcal{D}$-dissection.*

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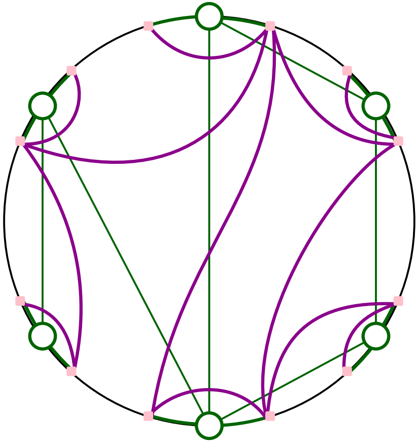

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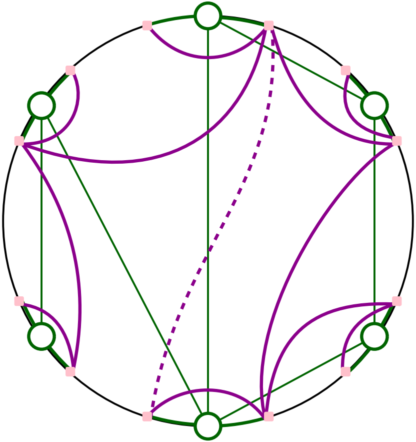

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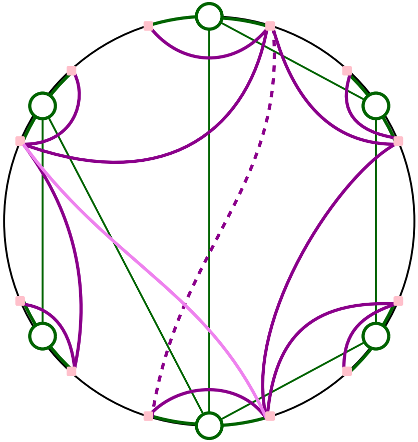

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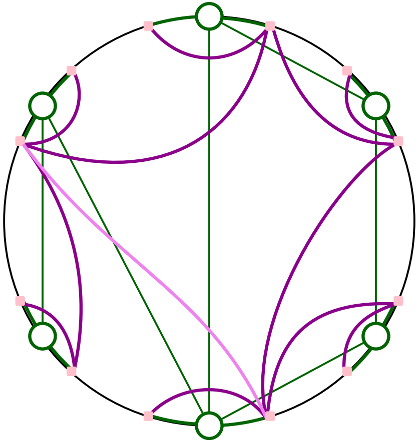

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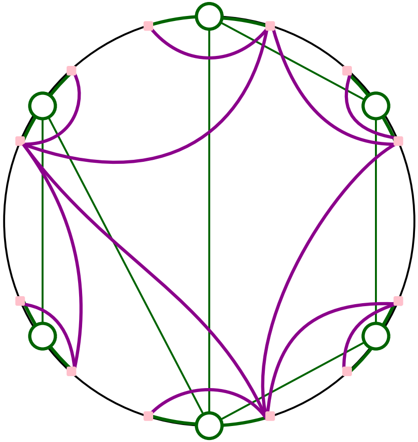

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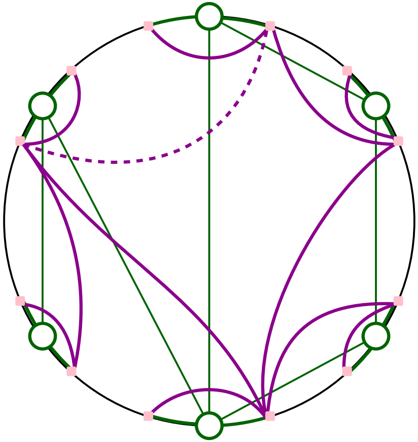

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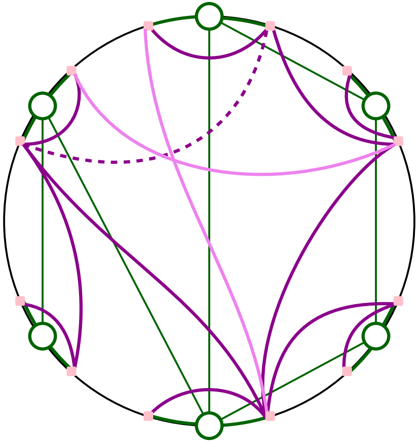

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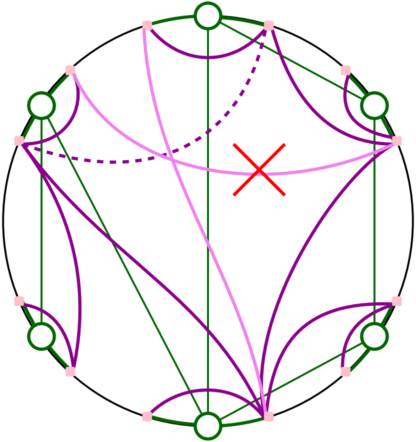

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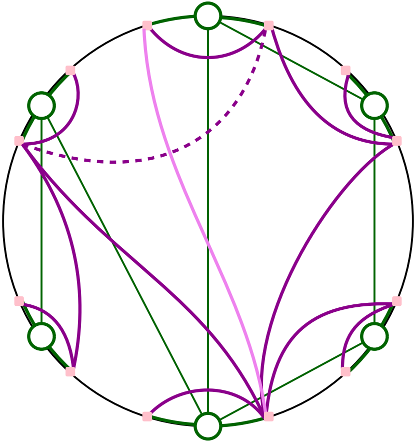

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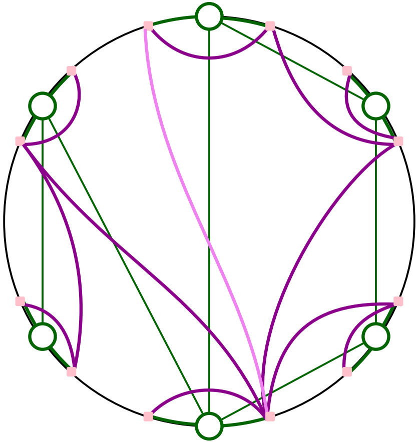

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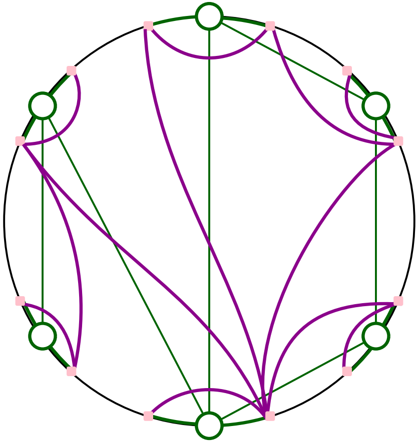

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Theorem (P–Pilaud–Plamondon)

The whole story generalises to oriented surfaces with boundaries, marked points (and punctures).

A category of accordions

Fix a (surface) dissection $\mathcal{D}$. We construct a Hom-finite, Krull–Schmidt additive category $\mathcal{A}$ whose indecomposable objects are the accordions.

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3. *Maximal $\mathcal{D}$-dissections coincide with the basic silting objects of $\mathcal{A}$.*

Flips are mutations

(a) Krull–Schmidt, Hom-finite, $\operatorname{proj} \mathcal{C} = \operatorname{add} P$.

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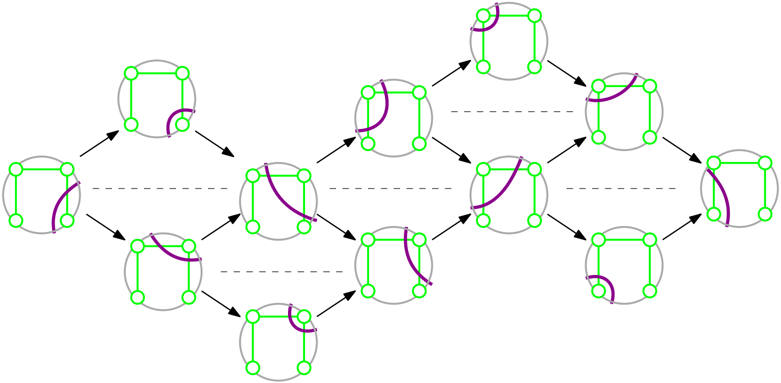
3. *There is a conflation $R_j \rightarrowtail E \twoheadrightarrow R_j^*$ xor $R_j^* \rightarrowtail E' \twoheadrightarrow R_j$ with $E, E' \in \operatorname{add}(\bigoplus_{i \neq j} R_i)$.*

Example



THANK YOU!