On m-cluster tilted algebras arising from a quiver of type $\mathbb{E}_p$

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Joint work with Natalia Bordino and Ulises Pallero, Universidad Nacional de Mar del Plata.

In this talk we are going to discuss m-cluster tilted algebras arising from a quiver of type $\mathbb{E}_p$. In particular we concentrate in the case $\mathbb{E}_6$. Following Thomas we recall the m-cluster category

$\mathcal{C}_m = D^b(\text{mod}H)/\tau^{-1}[m]$ and the fundamental domain $\mathcal{S}_m$ of $\mathcal{C}_m$.

We recall that, $\tilde{T}$ is an m-cluster tilting object if:

a) $\mathrm{Ext}^i_{\mathcal{C}_m(H)}(\tilde{T}, \tilde{T}) = 0$ for $i = 1, ..., m$.

b) The number of indecomposable classes of indecomposable summands of $\tilde{T}$ is equal to the number of isomorphic classes of simple H-modules.

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when $m = 1$, these are the cluster tilted algebras introduced by Buan, Marsh and Reiten.

We recall that a complex T is silting in $D^b(\text{mod } H)$ if:

(a) If $\text{Hom}_{D^b(modH)}(T, T[i]) = 0$ for all $i > 0$.

(b) The number of indecomposable summands of T is equal to the number of simple H-modules.

and T is tilting if also $\text{Hom}_{D^b(modH)}(T, T[i]) = 0$ for all $i < 0$.

Lemma. (Buan-Thomas-Reiten) There exists a one to one correspondence between silting complexes in $\mathcal{S}_m$ and m-cluster tilting objects in $\mathcal{C}_m$. That is, are equivalent,

1. T is a silting complex in $\mathcal{S}_m$.
2. $\tilde{T}$ is an m-cluster tilting object in $\mathcal{C}_m$.

We consider the hereditary algebra H:

$$\begin{array}{ccccccccc} 1 & \longleftarrow & 2 & \longleftarrow & 3 & \longleftarrow & 4 & \longleftarrow & 5 \\ & & & & \downarrow & & & & \\ & & & & 6 & & & & \end{array}$$

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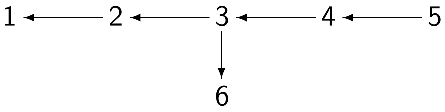

and we consider $m = 2$ and $m = 3$.

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Ulises Pallero has developed an algorithm to compute all the silting complexes in the fundamental domain.

This algorithm works for $\mathbb{E}_p$ and for every translation quiver having an additive function.

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Remark.

It seems that all m-cluster tilted algebras arising from a quiver of type $\mathbb{E}_6$ are given by the $1, 2, 3$-cluster tilted algebras of this kind.

Remark

The non connected 2,3-cluster tilted algebras arising from a quiver of type $\mathbb{E}_6$ are direct products of $1, 2, 3$-cluster tilted algebras arising from a quiver of type $\mathbb{A}$ and/or $\mathbb{D}$.

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The type Q is not well defined, there are m-cluster tilted algebras arising from quivers of $\mathbb{A}$ and $\tilde{\mathbb{A}}$ type, see Gubitosi's PhD Thesis, and our joint work with Fernández, García Elsener.

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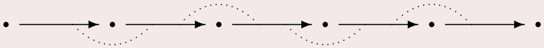
Here we show examples of m cluster tilted algebras arising from Dynkin quivers of different types, there exist m-cluster tilted algebras arising from a quiver of $\mathbb{E}_6$ and $\mathbb{A}$ type, or $\mathbb{D}$ type, and we also get m-cluster tilted algebras arising from a quiver of type $\mathbb{E}_6$ and Euclidean type $\tilde{\mathbb{D}}$.

The algebra



is a 2-cluster tilted algebra arising from a quiver of type $\mathbb{E}_6$ and also is a 5-cluster tilted algebra arising from a quiver of type $\mathbb{A}_6$.

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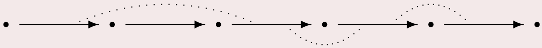
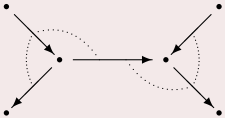


is 2-cluster tilted algebra arising from a quiver of type $\mathbb{E}_6$ and also a 4-cluster tilted algebra arising from a quiver of type $\mathbb{D}_6$.

The algebra



is a 2-cluster tilted algebra arising from a quiver of type $\mathbb{E}_6$ and also a 3-cluster tilted algebra arising from a quiver of type $\widetilde{\mathbb{D}_5}$.

Assem, Brüstle, Schiffler proved that 1-cluster tilted algebras A are given by trivial extensions, they showed that are relation extensions, $A = B \ltimes \mathrm{Ext}^2(DB, B)$ where B is a tilted algebra.

In a joint work with Fernández and Pratti we proved that f B is m-cluster tilted of tilting type Q then B is a trivial extension, is the higher relation extension studied by Assem, Gatica and Schiffler, $B = A \ltimes \mathrm{Ext}^{m+1}(DA, A)$ where A is an iterated tilted of type Q.

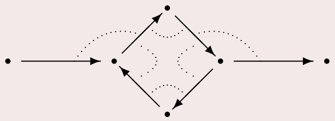
We prove now that the converse is also true.

Proposition

If B is the higher relation extension, $B = A \ltimes \text{Ext}^{m+1}(DA, A)$, where A is an iterated tilted of type Q then B is m-cluster tilted algebra of tilting type arising from a quiver of type Q.

If B is m-cluster tilted algebra not of tilting type then not necessarily B is a trivial extension of this kind.

Let A be the algebra



is 2-cluster tilted arising form a quiver of type $\mathbb{E}_6$ and there does not exist a triangular algebra B such that $A = B \ltimes \mathsf{Ext}^{m+1}(DB, B)$.