Pop-stack Sorting for Torsion Classes

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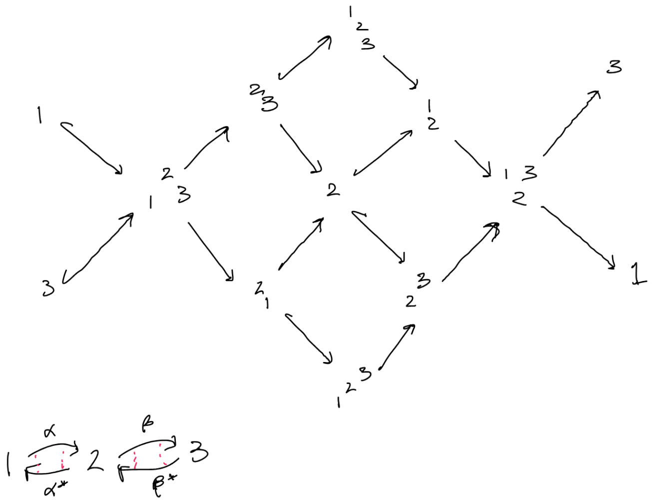
Running Example

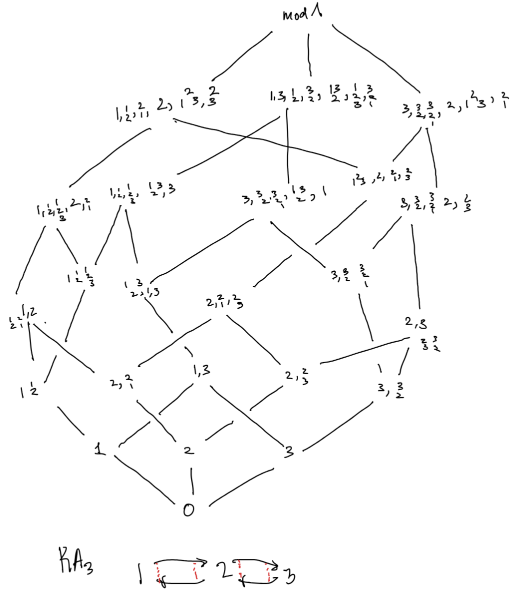
Let Λ be the gentle algebra defined taking the type A Dynkin diagram with arrows in both directions, and with the relations generated by killing all two cycles.

$$RA_3 = K\overline{Q}_A/(\alpha\alpha^*, \beta\beta^*, \alpha^*\alpha, \beta^*\beta)$$

Running Example



Torsion Classes



Brick Labeling

The lattice of torsion classes

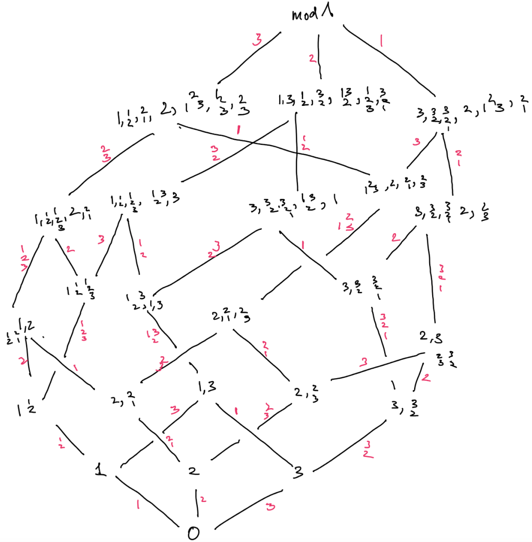
- We say that a brick S *labels* an upper cover relation $\mathcal{T} \lessdot \mathcal{T}'$ in the lattice torsΛ provided that $\mathcal{T}' = \mathrm{Filt}(\mathcal{T} \cup S)$. That is, $\mathcal{T}'$ is the closure of $\mathcal{T} \cup \{S\}$ under iterative extensions.

- The brick S is called a **minimal extending module**.

- Dually, a brick S *labels* a lower cover relation $\mathcal{T} \gtrdot \mathcal{T}''$ if S labels the corresponding relation $(\mathcal{T}'')^{\perp} \lessdot \mathcal{T}^{\perp}$ in the lattice of torsion free classes.

Running Example

Brick labeling



Pop-stack sorting

Notation

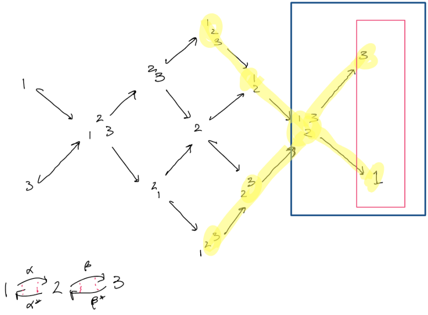
Let $\mathcal{D}$ is the set of bricks labeling its lower cover relations.

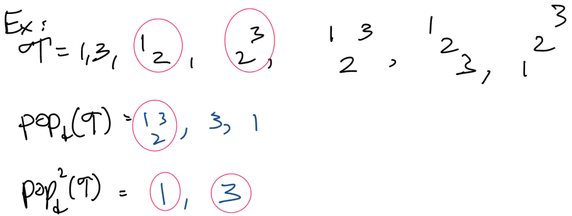
Definition

Given a torsion class $\mathcal{T}$ with lower covers labeled by bricks $\mathcal{D}$, define:

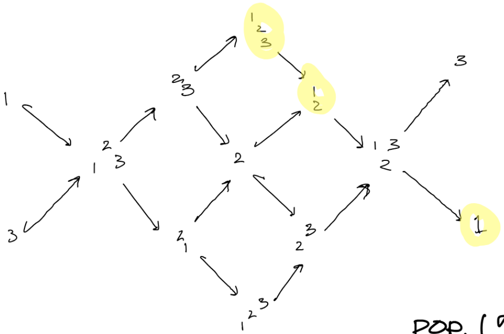
$$\mathrm{pop}^{\downarrow}(\mathcal{T}) = \mathcal{T} \cap {}^{\perp}D$$

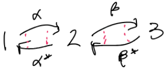
Running Example





Running Example





$\mathcal{T} = 1,\ 12,\ 123$

$\mathrm{pop}_{\downarrow}(\mathcal{T}) = 1,\ 12$

$\mathrm{pop}_{\downarrow}^{2}(\mathcal{T}) = 1$

$\mathrm{pop}_{\downarrow}^{3}(\mathcal{T}) = 0$

$\mathrm{pop}_{\downarrow}(\mathcal{T}) = \mathcal{T} \wedge {}^{\downarrow}\vartheta$

Image of pop-stack

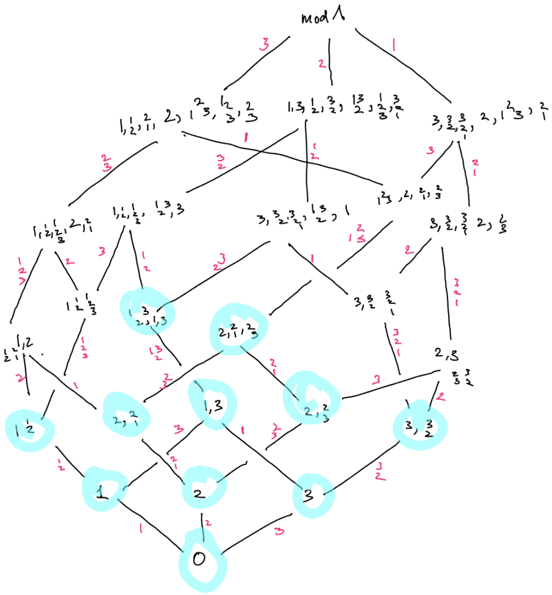


Image of pop-stack

Theorem [B., Defant, Hanson]

Suppose Λ is hereditary, and let $\mathcal{T} \in \mathsf{tors}(\Lambda)$. Then $\mathcal{T}$ is in the image of $\mathrm{pop}^{\downarrow}_{\mathsf{tors}\Lambda}$ if and only if both of the following hold:

1. $\mathrm{Ext}^1_\Lambda(X, X') = 0$ for all $X, X' \in \mathcal{D}(\mathcal{T})$.
2. There does not exist a nonzero projective module $P \in \mathcal{T}$.

Remark!

The theorem does not hold in general for the algebra RA_n.

Many many open questions!!

Let Λ be a τ-tilting finite.

- Compute the image of $\mathrm{pop}^{\downarrow}$.
- Find an upper bound for k, such that $\left(\mathrm{pop}^{\downarrow}\right)^{k}(\mathcal{T}) = 0$ for all torsion classes $\mathcal{T}$.
- Count the number of torsion classes which require the maximum number of iterations of $\mathrm{pop}^{\downarrow}$ in order to be "sorted".

Thank you!!

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