Flash Talks in Representation Theory V

January 10, 2025

Flash Talks in Representation Theory V is an online one-day conference hosted by NTNU. We present an overview of recent advances in representation theory through a series of 10-minute flash talks.

Thomas Brüstle

An exact structure approach to almost rigid modules

The recently introduced notion of almost rigid modules for type A is a weakening of the classical notion of rigid modules. The importance of this new notion stems from the fact that maximal almost rigid modules are in bijection with triangulations of a polygon. In this talk, we give a different realization of maximal almost rigid modules using a non-standard exact structure such that the maximal almost rigid modules are exactly the maximal rigid modules relative to the new exact structure.

arXiv:2410.04627 [pdf, ps, other] math.RT math.CT

An exact structure approach to almost rigid modules over quivers of type $\mathbb{A}$

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arXiv > math > arXiv:1912.02840

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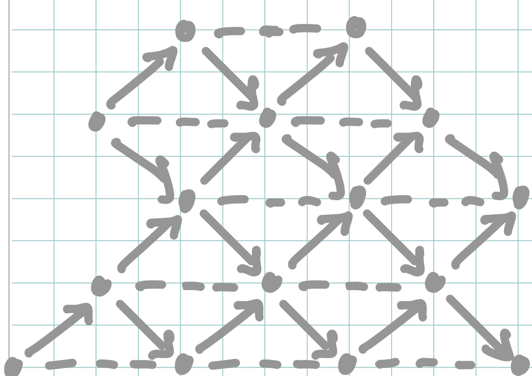
Cambrian combinatorics on quiver representations (type A)

Emily Barnard, Emily Gunawan, Emily Meehan, Ralf Schiffler

This paper presents a geometric model of the Auslander-Reiten quiver of a type A quiver together with a stability function for which all indecomposable modules are stable. We also introduce a new Catalan object which we call a maximal almost rigid representation. We show that its endomorphism algebra is a tilted algebra of type A. We define a partial order on the set of maximal almost rigid representations and use our new geometric model to show that this partial order is a Cambrian lattice.

Q type A_n quiver



$X, Y \in \Gamma_{kQ}$ Auslander-Reiten quiver

non-split s.e.s.

$0 \to X \to E \oplus E' \to Y \to 0$ (I)

$0 \to X \longrightarrow E \longrightarrow Y \to 0$ (II)

M basic kQ-module.

Definition: • M rigid if $\mathrm{Ext}^1_{kQ}(M,M)=0$

[E^3R '19] • M almost rigid if M does not have direct summands X, Y which admit a non-split s.e.s.

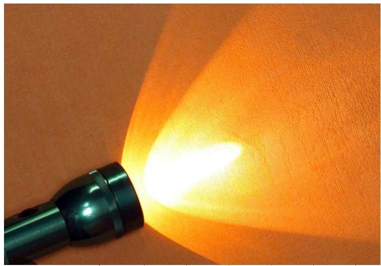
$$0 \to X \to E \oplus E' \to Y \to 0 \quad (I)$$

• M maximal almost rigid (MAR) if M is almost rigid, but no basic $M \oplus N$ is almost rigid

[E^3R '19]

$$\left\{ \text{MAR modules} \right\} \stackrel{1-1}{\longleftrightarrow} \left\{ \begin{matrix} \text{triangulations} \\ \text{of } n+1\text{-gon} \end{matrix} \right\}$$

mutation $\longleftrightarrow$ flip



Our interpretation:

almost rigid = rigid w.r.t. a subset of $\mathrm{Ext}^1(M,M)$

Definition: • An <u>exact structure</u> is an additive sub-bifunctor

$$\mathcal{E}(-,-) \subseteq \mathrm{Ext}^1_Q(-,-) \quad \text{which is } \underline{\text{closed}}$$

$$\left.\begin{array}{l} 0 \to X \xrightarrow{i} Y \to Z \to 0 \text{ in } \mathcal{E} \\ 0 \to Y \xrightarrow{j} U \to V \to 0 \text{ in } \mathcal{E} \end{array}\right\} \Rightarrow \exists \; 0 \to X \xrightarrow{j \circ i} U \to W \to 0 \text{ in } \mathcal{E}$$

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Exact structures allow to define $\mathcal{E}$-projectives, $\mathcal{E}$-resolutions, $\mathcal{E}$-derived category ...

• M is $\mathcal{E}$-rigid if $\mathcal{E}(M,M) = 0$

Theorem [BHRS'24]

(1) $\exists$ exact structure $\mathcal{E}_\diamond \subseteq \mathrm{Ext}^1_Q$ such that

M is $\mathcal{E}_\diamond$-rigid $\iff$ M is almost rigid

(2) M is $\mathcal{E}_\diamond$-tilting $\iff$ M is maximal almost rigid

(3) $(\mathrm{mod}\, KQ, \mathcal{E}_\diamond)$ is an 0-Auslander category

(Gorsky, Nakaoka, Palu: mutation of $\mathcal{E}_\diamond$-tilting modules, ...)

(i) $\mathcal{E}_\diamond$-global dimension of kQ is ≤ 1

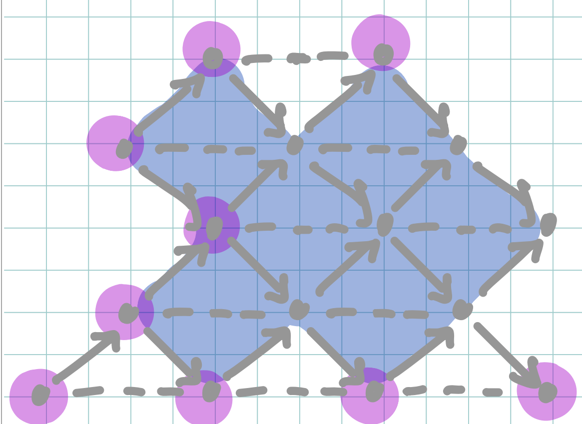
(ii) $\mathcal{E}_\diamond$-dominant dimension of kQ is ≤ 1

($\forall P \in \mathrm{proj}(\mathcal{E}_\diamond)$ $\exists$ $0 \to P \to U \to I \to 0$
with U proj-inj, I inj relative to $\mathcal{E}_\diamond$)

Construction of $\mathcal{E}_\diamond$:

By [Auslander, Solberg '93], it is sufficient to specify which AR-sequences belong to $\mathcal{E}_\diamond$

($\Leftrightarrow$ which modules are $\mathcal{E}_\diamond$-projective)



$\mathcal{E}_\diamond$ given by the AR-sequences with 2 middle terms

Then, for $X, Y \in \mathcal{T}_{kQ}$: $\mathcal{E}_\diamond(X,Y) = \{ \eta \mid \eta \text{ split or } \eta \text{ has 2 middle terms} \}$

<u>Rmk</u>: Not valid for type D quivers

Example 5.1. Let Q be the quiver

$$\begin{tikzcd}[row sep=small]
 & & 3 \\
1 \arrow[r, "\alpha"] & 2 \arrow[ur, "\beta"] \arrow[dr, "\gamma"'] & \\
 & & 4
\end{tikzcd}$$

$$\begin{tikzcd}
3 \arrow[dr] & & \begin{smallmatrix} 2 \\ 4 \end{smallmatrix} \arrow[ll, dashed] \arrow[dr] & & \begin{smallmatrix} 1 \\ 2 \\ 3 \end{smallmatrix} \arrow[ll, dashed] \arrow[dr] & & \\
 & \begin{smallmatrix} & 2 & \\ 3 & & 4 \end{smallmatrix} \arrow[ur] \arrow[r] \arrow[dr] & \begin{smallmatrix} & 1 & \\ & 2 & \\ 3 & & 4 \end{smallmatrix} \arrow[r] & \begin{smallmatrix} & 1 & \\ 2 & & 2 \\ 3 & & 4 \end{smallmatrix} \arrow[ur] \arrow[r] \arrow[dr] & 2 \arrow[r] & \begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \arrow[r] & 1 \\
4 \arrow[ur] & & \begin{smallmatrix} 2 \\ 3 \end{smallmatrix} \arrow[ll, dashed] \arrow[ur] & & \begin{smallmatrix} 1 \\ 2 \\ 4 \end{smallmatrix} \arrow[ll, dashed] \arrow[ur] & &
\end{tikzcd}$$

We consider the diamond exact sequence ξ :

$$0 \longrightarrow \begin{smallmatrix} & 2 & \\ 3 & & 4 \end{smallmatrix} \longrightarrow \begin{smallmatrix} 2 \\ 4 \end{smallmatrix} \oplus \begin{smallmatrix} & 1 & \\ & 2 & \\ 3 & & 4 \end{smallmatrix} \longrightarrow \begin{smallmatrix} 1 \\ 2 \\ 4 \end{smallmatrix} \longrightarrow 0.$$

By taking the push-out of ξ along $f : \begin{smallmatrix} & 2 & \\ 3 & & 4 \end{smallmatrix} \to \begin{smallmatrix} 2 \\ 3 \end{smallmatrix}$, we obtain the short exact sequence:

$$0 \longrightarrow \begin{smallmatrix} 2 \\ 3 \end{smallmatrix} \longrightarrow \begin{smallmatrix} & 1 & \\ 2 & & 2 \\ 3 & & 4 \end{smallmatrix} \longrightarrow \begin{smallmatrix} 1 \\ 2 \\ 4 \end{smallmatrix} \longrightarrow 0$$

which is not a diamond exact sequence. Therefore, the diamond exact sequences do not form an exact structure for type $\mathbb{D}$ quivers.

감사합니다
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