ANALYSIS NEAR THE POLE

Day	Time	Program
MONDAY	09:00-12:00	Bondarenko
	12:00	*Lunch*
	15:00-18:00	*Cultural-historical sightseeing*
	19:00	*Dinner*
TUESDAY	09:00-12:00	Saksman
	12:00	*Lunch*
	14:00-14:30	Junnila
	14:40-15:10	Mahatab
	15:10	*Coffee break*
	15:30-16:00	Perfekt
	16:10-16:40	Haimi
	19:00	*Dinner*
WEDNESDAY	09:00-11:30	Malinnikova
	11:30	*Lunch*
	12:45-19:30	*Boat trip*
	20:30	*Dinner*
THURSDAY	09:00-12:00	Nazarov
	12:00	*Lunch*
	14:00-14:30	Glücksam
	14:40-15:10	Buckley
	15:10	*Coffee break*
	15:30-16:00	Heinävaara
	16:10-16:40	Seo
	19:00	*Conference dinner*
FRIDAY	09:00-12:00	Makarov
	12:00	*Lunch*
	14:00-14:30	Kiro
	14:40-15:10	Wennman
	15:10	*Coffee break*
	15:30-16:00	Huusko
	19:00	*Dinner*

A. Extreme values of the Riemann zeta function

Speaker. Andriy Bondarenko.

Abstract. We prove that for every $c < 1$ there exists arbitrarily large T with

$$|\zeta(1/2 + iT)| > \exp\left(c\sqrt{\log T \log\log\log T / \log\log T}\right).$$

This improves classical results by Montgomery, Balasubramanian–Ramachandra, and Soundararajan. We will discuss the main components of the proof: Soundararajan's resonance method, multiplicative functions, and convolution formulas for the Riemann zeta function. The relation to the greatest common divisor sums will be also shown.

B. Introduction to multiplicative chaos with applications to Dirichlet series

Speaker. Eero Saksman.

Abstract. We begin by providing an elementary introduction to Gaussian multiplicative chaos. Next we explain how it is related to the statistics of the Riemann zeta function. Finally, we also aim to describe how chaos philosophy appears in the recent resolution of Helson's conjecture due to Adam Harper.

C. A guided tour to harmonic functions on integer lattices

Speaker. Eugenia Malinnikova.

Abstract. This is an easy and relaxing walk along a side road accessible for everyone. We enter the discrete world and, after a very short introductory tour, we reach an observation point with a view of our main destination: an improved version of the Liouville theorem. A stop at the observation point is designed to prepare for the further journey. The path to the sight consists of two parts with very different terrains. The scenery along the first part will be familiar, it reminds the continuous world. The visible road is however blocked and we will take a small detour to overcome the obstacle. On the second part the surroundings are new and, hopefully, you will enjoy the beauty of the discrete world.

The tour is based on a joint work with L. Buhovsky, A. Logunov and M. Sodin.

D. Sign changes of stationary Gaussian processes with spectral gap.

Speaker. Fedor Nazarov.

Abstract. We shall discuss the following question: Let f be a stationary Gaussian process on the line whose spectral measure μ satisfies the gap condition $\mu([-\delta, \delta]) = 0$. How large can the probability that $f \geq 0$ on $[0, L]$ be?

E. Etudes for the inverse spectral problem

Speaker. Nikolai Makarov.

Abstract. Following de Branges' approach, I will derive formula solutions and give some examples in the half-line case. The talk will be mostly expository and based on the joint work with Alex Poltoratski.

1. Measurably entire functions and their growth

Speaker. Adi Glücksam.

Abstact. Let (X, B, P) be a standard probability space. Let $T\colon \mathbb{C} \to PPT(X)$ be a free action of the complex plane on the space (X, B, P). We say that the function $F\colon X \to \mathbb{C}$ is measurably entire if it is measurable and for P-a.e x the function $F_x(z) := F(T_z x)$ is entire. B. Weiss showed in '97 that for every free $\mathbb{C}$ action there exists a non-constant measurably entire function. In the talk I will present upper and lower bounds for the growth of such functions.

The talk is partly based on a joint work with L. Buhovsky, A. Logunov, and M. Sodin.

2. A central limit theorem for fluctuations in Polyanalytic Ginibre ensembles

Speaker. Antti Haimi.

Abstract. We study fluctuations of linear statistics in Polyanalytic Ginibre ensembles, a family of point processes describing planar free fermions in a uniform magnetic field at higher Landau levels. Our main result is asymptotic normality of fluctuations, extending a result of Rider and Virag. As in the analytic case, the variance is composed of independent terms from the bulk and the boundary. Our methods rely on a structural formula for polyanalytic polynomial Bergman kernels which separates out the different pure q-analytic kernels corresponding to different Landau levels. The fluctuations with respect to these pure q-analytic Ginibre ensembles are also studied, and a central limit theorem is proved. The results suggest a stabilizing effect on the variance when the different Landau levels are combined together.

The presentation is based on joint work with Aron Wennman.

3. On planar orthogonal polynomials

Speaker. Aron Wennman.

Abstract. Let $\{P_{m,0}, P_{m,1}, P_{m,2}, \ldots\}$ denote the sequence of orthonormal polynomials that result from applying the Gram-Schmidt procedure to the monomials sequence $\{1, z, z^2, \ldots\}$, with respect to the inner product of $L^2\big(e^{-2mQ(z)}dA(z)\big)$. We will discuss an asymptotic expansion for $P_{m,n}$, as $m, n \to +\infty$ with $n \approx m\tau$, which well describes the behaviour of these polynomials outside a certain emergent compact set $\mathcal{S}_\tau$ — the *droplet*.

Our proof relies on the construction of an almost orthonormal family $\{F_{m,n}\}$ of holomorphic functions on $\mathcal{S}_\tau^c$, with the correct polynomial growth at infinity — so called *quasipolynomials* — and a polynomialization scheme using Hörmander-type $\bar{\partial}$-estimates.

We will discuss the relation of our work to classical strong asymptotics of planar orthogonal polynomials by Szegő, Carleman and Suetin.

This is a report of ongoing joint work with Håkan Hedenmalm.

4. The punctual image and summation problems in Beurling classes of smooth functions

Speaker. Avner Kiro.

Abstract. The talk will be about two problems in the theory of Beurling and Carleman classes of smooth functions. The first one is to describe the image of a Beurling or Carleman class under Borel's map $f \mapsto (f^{(n)}(0))_{n\geq 0}$. The second one is how to construct a function in Beurling or Carleman class with prescribed Taylor coefficients. In the talk, I will present solutions to both problems in some Beurling and Carleman classes (both quasianalytic and non-quasianalytic).

5. Non-Gaussian multiplicative chaos

Speaker. Janne Junnila.

Abstract. Random distributions (generalized functions) known as Gaussian multiplicative chaos appear as limits of various random objects related to e.g. random matrices and number theory. In this talk we introduce non-Gaussian generalizations of these distributions and discuss their existence and other basic properties.

6. The increment of the argument for the GEF

Speaker. Jerry Buckley.

Abstract. The Gaussian entire function (GEF) is a random entire function, characterised by a certain invariance with respect to isometries of the plane. We study the increment of the argument of the GEF along planar curves. We introduce an inner product on these curves, that we call the signed length, which describes the limiting covariance of the increment. We also establish asymptotic normality of fluctuations.

Joint work with M. Sodin.

7. On Becker's univalence criterion

Speaker. Juha-Matti Huusko.

Abstract. We discuss locally univalent functions f analytic in the unit disc $\mathbb{D}$ of the complex plane such that

$$\left|\frac{f''(z)}{f'(z)}\right|(1-|z|^2) \leq 1 + C(1-|z|), \quad z \in \mathbb{D},$$

for some $0 < C < \infty$. If $C \leq 1$, then f is univalent by Becker's univalence criterion. We discover that for $1 < C < \infty$ the function f remains to be univalent in certain horodiscs. Sufficient conditions which imply that f is bounded, belongs to the Bloch space or belongs to the class of normal functions, are discussed. Moreover, we consider generalizations for locally univalent harmonic functions.

Joint work with Toni Vesikko.

8. Extreme values of the Riemann zeta function on the 1-line

Speaker. Kamalakshya Mahatab.

Abstract. We shall prove that there are arbitrarily large values of t for which

$$|\zeta(1+it)| \geq e^{\gamma}(\log_2 t + \log_3 t) + O(1).$$

Here ζ denotes the Riemann zeta function.

This is a joint work with Christoph Aistleitner and Marc Munsch.

9. Helson matrices: Boundedness, moment problems, and finite rank

Speaker. Karl-Mikael Perfekt.

Abstract. A Helson matrix (also known as a multiplicative Hankel matrix), is an infinite matrix of the form $M(\alpha) = \{\alpha(nm)\}_{n,m=1}^{\infty}$, where α is a sequence of complex numbers. As linear operators on $\ell^2(\mathbb{N})$, Helson matrices generalize Hankel matrices $\{\beta(j+k)\}_{j,k=0}^{\infty}$.

Helson initiated the study of their boundedness, but the theory has not yielded a characterization. However, when a Helson matrix is positive semidefinite it may be realized as the moment sequence of a measure μ on $\mathbb{R}^{\infty}$, assuming that α does not grow too fast. This gives a description of the bounded non-negative Helson matrices in terms of Carleson measures for the Hardy space of countably many variables.

The model example of a Helson matrix, the multiplicative Hilbert matrix, falls into this class. Its spectrum has now been fully determined, a first step in developing the spectral theory of Helson matrices. Furthermore, we characterized the Helson matrices of finite rank, giving an analogue of Kronecker's theorem in this context.

Based on joint work with Alexander Pushnitski.

10. Matrix monotone functions

Speaker. Otte Heinävaara.

Abstract. *Matrix monotonicity* is the notion one gets by combining functional calculus and Loewner order on Hermitian matrices. We discuss classic and recent characterizations for the the class of matrix monotone functions.

11. Bulk scaling limits of random normal matrix ensembles near singularities

Speaker. Seong-Mi Seo.

Abstract. In this talk, I will present microscopic properties of the eigenvalues of random normal matrices. Specifically, this talk will focus on the random normal matrix model with a singularity in the interior of the spectrum. I will discuss the existence and uniqueness of the microscopic density of eigenvalues near a singularity and describe how the rescaled Ward's identity can be used to prove the universality.

This is based on joint work with Yacin Ameur.