Complex Geometry and the Cauchy-Riemann Equation

Oslo, August 22 - 26 2016

PROGRAM AND ABSTRACTS

This conference is part of the special year on Several Complex Variables and Complex Dynamics at the Centre for Advanced Studies in Oslo

During the conference we will celebrate the 70'th birthday of John Erik Fornæss

Centre for Advanced Study
at the Norwegian Academy of Science and Letters

Senter for grunnforskning
ved Det Norske Videnskaps-Akademi

	Monday August 22	Tuesday August 23	Wednesday August 24	Thursday August 25	Friday August 26
9:00 - 9:30	Registration				
9:30 - 10:20		Bo Berndtsson	László Lempert	Michael Range	Takeo Ohsawa
10:20 - 10:30	Scientific Director				
10:30 - 11:20	Joe Kohn	Jürgen Leiterer	Stefan Nemirovski	Jean Ruppenthal	Yum-Tong Siu
11:20 - 12:00	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
12:00 - 12:50	Peter Ebenfelt	Zbigniew Błocki	Xu Wang	Emil Straube	
13:00 - 14:30	Lunch break	Lunch break	Lunch break	Lunch break	
14:30-15:20	Nessim Sibony	Bo-Yong Chen	Mei-Chi Shaw	Free afternoon	
15:30 - 16:20	Christine Laurent-Thiébaut				
16:30 - 18:30	Wine and Cheese				
19:00 - ??				Banquet	

Joseph Kohn

Title: Generalized CR manifolds

Abstract: Let Ω be an n-dimensional, compact, complex manifold with a smooth, connected boundary $b\Omega$. Denote by $\mathcal{H}_b$ the subspace of $C^\infty(b\Omega)$ consisting of boundary values of holomorphic functions. We consider pseudodifferential operators on $b\Omega$ that annihilate $\mathcal{H}_b$. We say that these operators define a CCR structure on $b\Omega$ (this includes also the case when $n = 1$). For example if L is a vector field in $T^{(1,0)}(\bar{\Omega})$ then we construct the pseudodifferential operator $\bar{L}^{\#}$ as follows. If $f \in C^\infty(b\Omega)$ let Ef be the harmonic function in $C^\infty(\bar{\Omega})$ whose restriction to $b\Omega$ equals f. Then we define $\bar{L}^{\#}f$ to be the restriction of $\bar{L}f$ to $b\Omega$. Using this operator we can prove new microlocal estimates on CR functions. We show that the symbol of $\bar{L}^{\#}$ depends only on the CR structure on $b\Omega$. These considerations lead to a definition of abstract CCR manifolds and opens an approach to the study of local and global embeddings of CCR manifolds and their underlying CR manifolds.

Peter Ebenfelt

Title: The Bergman and Szegő kernels and the Obstruction Function in strictly pseudoconvex domains in $\mathbb{C}^2$

Abstract: We will consider bounded strictly pseudoconvex domains Ω in $\mathbb{C}^2$. The obstruction function $b\eta_1$ on $\partial\Omega$ is the lowest order term in the singularity at the boundary of the Cheng-Yau solution to the Dirichlet problem for Fefferman's Monge-Ampere operator in Ω. By the work of Graham and Hirachi-Komatsu-Nakzawa, this function is also the restriction to the boundary of the log term in the Bergman and Szegő kernels in Ω. We shall discuss the condition $b\eta_1 = 0$ on $\partial\Omega$; in particular, if Ω also has transversal symmetry, then we will show that if $b\eta_1 = 0$ globally, then $\partial\Omega$ is locally spherical. We note that it is not true that $b\eta_1 = 0$ locally implies that $\partial\Omega$ is locally spherical.

Nessim Sibony

Title: Levi Problem and Pfaff Systems

Abstract: Let U be a relatively compact pseudo-convex open set with smooth boundary in a complex manifold M. The question is wether U is Stein. There are famous counter-examples due to Grauert, and partial positive results due to Hirschowitz, Greene-Wu in particular. There is a survey by Siu on the Levi problem and a more recent one by Ohsawa.

I will give conditions which imply that U is Stein. They are related to the integrability of a natural Pfaff System. I will also discuss in the above setting, the existence of bounded strictly psh exhaustion functions, with useful estimates. If time permits, I will describe the geometry of pseudo-convex non-Stein domains and develop a foliated version of the above results.

The approach is related to my recent paper: Pfaff systems, currents and hulls (ArXiv).

Christine Laurent-Thiébaut

Title: Solving the Cauchy-Riemann equation with prescribed support

Abstract: Let Ω be an open subset of a complex manifold X, we define the Dolbeault cohomology groups on X with prescribed support in the closure of Ω as the quotient of the kernel of the Cauchy-Riemann operator in the space of smooth forms, L^2 forms or currents with support in the closure of Ω by the range of the Cauchy-Riemann operator on the space of smooth forms, L^2 forms or currents with support in the closure of Ω. We first prove that the Dolbeault isomorphism fails to hold for such groups. These groups are used to characterized pseudoconvex domains with pseudoconvex holes depending on the regularity of the boundary of the holes. (This is a joint work with Siqi Fu and Mei-Chi Shaw.)

Bo Berndtsson

Title: Symmetries on the space of Kähler metrics

Abstract: The Mabuchi space associated to a Kähler class on a compact manifold is the space of all Kähler forms in that class. It has a structure of an infinite dimensional Riemannian manifold that formally satisfies the requirements of a symmetric space. We will construct 'explicit' local symmetries around any real analytic Kähler form in the space. The construction is based on work of Lempert and can be seen as a generalized Legendre transform. (This is joint work with Dario Cordero-Erausquin, Bo'az Klartag and Yanir Rubinstein.)

Jürgen Leiterer

Title: Local and global similarity of holomorphic matrices

Abstract: R. Guralnick (Linear Algebra Appl., 85-96, 1988) proved: If two holomorphic matrices on a non-compact connected Riemann surface are locally holomorphically similar, then they are globally holomorphically similar. Actually, he proved a more general theorem for certain Bezout rings, and then applied this to the ring of global holomorphic functions on a non-compact connected Riemann surface. It seems that this proof cannot be generalized to non-smooth one-dimensional Stein spaces, and also not to smooth higher dimensional Stein spaces, because then the ring of global holomorphic functions is not Bezout. Using other methods, we obtain: (1) On one-dimensional Stein spaces, local holomorphic similarity implies global holomorphic similarity. (2) On arbitrary Stein spaces, local holomorphic similarity together with $\mathcal{C}^\infty$ similarity implies global holomorphic similarity.

The first step in the proof is the proof of the fact that the Jordan structure of a holomorphic matrix "does not jump" outside a nowhere dense analytic set. In the smooth case, this was earlier obtained by H. Baumgärtel (Birkhäuser 1985).

A counterexample shows that, in (2), $\mathcal{C}^\infty$ cannot be replaced by $\mathcal{C}^0$.

Other counterexamples show that local continuous similarity is not sufficient for local holomorphic similarity, even not on one-dimensional analytic sets. However, on Riemann surfaces, it is.

Zbigniew Błocki

Title: Geodesics in the spaces of Kähler metrics and volume forms

Abstract: We discuss optimal regularity of geodesics in the space of Kähler metrics of a compact complex manifold, as well as the space of volume forms on a compact Riemannian manifold. They are solutions of nonlinear degenerate elliptic equations: homogeneous complex Monge-Ampere equation and an equation introduced by Donaldson, respectively. The highest regularity one can expect is $\mathcal{C}^{1,1}$.

Bo-Yong Chen

Title: Parameter dependence of the Bergman kernels

Abstract: Let $\{\Omega_t : -1 < t < 1\}$ be a family of bounded pseudoconvex domains and $\varphi_t \in PSH(\Omega_t)$. Let $K_t(z, w)$ denote the Bergman kernel with weight φ_t on Ω_t. We study the continuity and Hölder continuity of $K_t(z, w)$ in t. Several applications are given, including a new $\bar{\partial}$ proof of the openness theorem.

László Lempert

Title: Noncommutative potential theory

Abstract: Traditional potential theory is the study of the Laplace operator, (pluri)subharmonic functions, etc. The geometric content of the Laplace operator is curvature of hermitian metrics on holomorphic line bundles. Accordingly, noncommutative potential theory is the study of curvature in holomorphic vector bundles of higher rank, in the spirit of traditional potential theory.

The talk will discuss some results in this vein: maximum principles, Dirichlet problems, mean value properties.

Stefan Nemirovski

Title: Two complex variables

Abstract: Complex analysis in two dimensions has many special features. The talk will concentrate on known results and open problems related to rigidity phenomena in low dimensional symplectic, contact and differential topology.

Xu Wang

Title: Curvature restrictions on a manifold with a flat Higgs bundle

Abstract: We shall study the curvature restrictions on a manifold with a flat Higgs bundle. We shall show that the base manifold of a flat admissible Higgs bundle is Kähler, which generalizes a result of Lu. We shall also prove a semi-negative curvature property for a manifold with a flat admissible Higgs bundle, which can be seen as a generalization of a result of Griffiths-Schmid (on the curvature property of the Hodge metric associated to a variation of Hodge structure).

Mei-Chi Shaw

Title: The Diederich-Fornæss exponent and the $\bar{\partial}$-equation

Abstract: Diederich and Fornæss showed in 1977 that for any bounded pseudoconvex domain Ω with C^2 boundary in a Stein manifold, there exist a positive constant η and a defining function r such that $\hat{r} = -(-r)^\eta$ is plurisubharmonic on Ω. In this talk we will discuss the consequences of such bounded plurisubharmonic exhaustion functions and its relations to solving the $\bar{\partial}$-equation. We will also report some recent new results on the L^2 closed range property for $\bar{\partial}$ on an annulus between two pseudoconvex domains, when the inner domain is not smooth. In particular, we show the Hausdorff property of the L^2 Dolbeault cohomology group on a domain between a ball and a bi-disc, the so-called Chinese Coin problem. Our methods also give the Sobolev W^1-estimates for the $\bar{\partial}$-equations on non-smooth domains, including certain product domains or intersection of smooth bounded pseudoconvex domains (joint work with Siqi Fu, Debraj Chakrabarti and Christine Laurent-Thiébaut).

R. Michael Range

Title: Pseudoconvexity: Highlights, Problems, and Some Results

Abstract: After a brief review of the history of pseudoconvexity and of some of the major results that have been proved on *strictly* pseudoconvex domains beginning in the 1960s, we turn to the general case of weakly pseudoconvex domains. While some deep and important results have been obtained in the finite type case, the general case still remains quite mysterious, with significant unresolved questions. We discuss some possible approaches that might help to gain a better understanding of the general case. We conclude with some positive results on weakly pseudoconvex domains that are based on a non-holomorphic modification of the familiar integral kernels on strictly pseudoconvex domains. In particular, we discuss a pointwise a-priori estimate that is an analogue of the classical basic estimate in the L^2 theory of the $\bar{\partial}$-Neumann problem, and outline potential applications.

Jean Ruppenthal

Title: About regularity of the Cauchy-Riemann equations at isolated singularities

Abstract: There has been a lot of research activity around the Cauchy-Riemann equations on singular complex spaces in the past thirty years. In this talk, I will give a brief overview over the main methods which were used in this field. A special emphasis will be put on the contribution of John Erik Fornaess, and the lines of research influenced by his work.

To keep the presentation concise, we will focus on isolated singularities, where - not surprisingly - the Cauchy-Riemann equations are best understood. Very recently, it turned out, that canonical singularities in the sense of the minimal model program are particularly tame with respect to the Cauchy-Riemann equations. If time allows, I will present some recent results with M. Andersson, R. Lärkäng, H. Samuelsson Kalm and E. Wulcan in this direction, where we use the integral

formulas of Andersson-Samuelsson to construct well-behaved $\bar{\partial}$-homotopy formulas at canonical surface singularities.

EMIL STRAUBE

Title: CR submanifolds of hypersurface type: geometry and estimates for $\overline{\partial}_M$

Abstract: We first discuss some geometric ideas concerning the submanifolds in the title, which are used in the remainder of the talk. Then we set up the L^2-theory of the complex Green operators and look at the question of when they are compact. Finally, we study Sobolev estimates for these operators. This talk presents a condensed version of a recent survey written jointly with Severine Biard.

TAKEO OHSAWA

Title: A variant of Hörmander's estimate and application to an approximation problem

Abstract: Let (M, φ) be a weakly 1-complete manifold of dimension n. Namely, we consider a complex manifold M of dimension n equipped with a C^∞ plurisubharmonic exhaustion function φ. Let E be a holomorphic vector bundle over M and let $H^{p,q}(M, E)$ denote the $\bar{\partial}$-cohomology group of type (p, q) with values in E. Let h be a fiber metric of E and let Θ_h denote the curvature form of h. In [N-R], Nakano and Rhai showed:

Theorem 1. *If Θ_h is Nakano positive outside $M_c := \{x \in M; \varphi(x) < c\}$,*
(1) $\forall q \geq 1$, $\dim H^{n,q}(M, E) < \infty$,
(2) $\forall q \geq 1$, *the restriction homomorphism $H^{n,q}(M, E) \to H^{n,q}(M_c, E)$ is bijective*
and
(3) *The image of the restriction homomorphism $H^{n,0}(M, E) \to H^{n,0}(M_c, E)$ is dense.*

Restricting ourselves to (3), we considered its generalizations and variants, and noticed that a variant of an L^2 estimate for the $\bar{\partial}$ operator in [H] implies the following approximation theorem.

Theorem 2. *Let (M, ω) be a Hermitian manifold of dimension n, let (E, h) be a Hermitian holomorphic vector bundle over M, and let D be a bounded pseudoconvex domain with C^∞ smooth boundary such that $\overline{D}$ admits a weakly 1-complete neighborhood basis. If (E, h) is Nakano positive on a neighborhood of ∂D, every element of $L^{n,q}(D, E, \omega, h) \cap \mathrm{Ker}\bar{\partial}$ can be approximated arbitrarily well with respect to the L^2 norm on D by $\bar{\partial}$-closed (n, q)-forms defined on neighborhoods of $\overline{D}$.*

References:

[N-R] Nakano, S. and Rhai, T.-S., *Vector bundle version of Ohsawa's finiteness theorems,* Math. Japon. **24** (1979/80), no. 6, 657-664.

[H] Hörmander, L., L^2 *estimates and existence theorems for the $\bar{\partial}$ operator*, Acta Math. **113** (1965), 89-152.

Yum-Tong Siu

Title: Multiplier Ideal Sheaves for General Systems of Partial Differential Equations

Abstract: Joe Kohn in 1979 introduced the techniques of multiplier ideal sheaves to blaze a trail in applying algebraic geometric methods to study the regularity of the Kohn solution of the complex Neumann problem for weakly pseudoconvex domains. In this talk we will present the generalization of the techniques of multiplier ideal sheaves to general systems of partial differential equations. We will also discuss the motivations and the roles of regularity questions and multiplier ideal sheaves in the broader context of problems in algebraic and complex geometry.