1.1. 2) Most of you have obtained correct rejection region

$$R = \left\{ x \;\middle|\; f(x|0) < \underbrace{\frac{\pi(0)(1-k)}{\pi(1)}}_{\text{we denote it } k'} f(x|1) \right\}$$

3) One of possible solutions:

We observe that

$$\beta(0) = \underbrace{\int_R f(x|0)}_{\substack{\text{If } k' \leq 1, \\ \text{compare it with} \\ \int_R k' f(x|1)}} = \underbrace{1 - \int_{R^c} f(x|0)}_{\substack{\text{If } k' > 1 \\ \text{compare it with} \\ 1 - \int_{R^c} k' f(x|1)}}$$

And use 1.1.2.

1.2.4. Yes.

Many misinterpreted that k in 1.2. and 1.1. was the same and therefore justified the answer 'no'. This gave full score too.

1.3.3. Note that Karlin-Rubin theorem as formulated by Th. 8.3.17 in Casella & Berger works only for non-decreasing likelihood ratio in t.

1.3.4.

It is a UI test (check definition)

Under assumptions of both 1.3.1 and 1.3.3 — both most powerful and unbiased, see Keener (2010), Ch. 12

Under assumptions of only 1.3.3 — no, it may be not most powerful although being unbiased and may be neither most power nor unbiased (find examples!)

1.3.5 See lectures

1.3.7 Note that there is not MLR in x, only MLR in t

2.1.6 See 1.3.4 and Ch. 12 in Keener (2010)

2.2.3 Find γ_1 and γ_2 from comparing the loss in 2.2.3 and 2.2.1

2.2.4

$$\rho = E(|S|)\gamma_1 + P(\mu \notin S)\gamma_2 \overset{\text{(Pratt theorem)}}{=} \gamma_1 \int \underbrace{P(\mu \in S)}_{\substack{\text{note that} \\ \text{this varies with } \mu \\ \text{and thus cannot be} \\ \text{just replaced with } (1-\alpha)}} d\mu + \alpha \cdot \gamma_2$$

2.2.5

Set into 2.2.4 γ_1 and γ_2 from 2.2.3 (and 2.1.3),

Check that $\dfrac{E(|S|)}{\sigma} = \begin{cases} \dfrac{2 z_{\alpha/2}}{\sqrt{n}}, & \sigma \text{ known} \\ \dfrac{2 t_{\frac{\alpha}{2}, n-1}}{\sqrt{n}} \quad E\left(\dfrac{\hat{\sigma}}{\sigma}\right) = \dfrac{2 t_{\frac{\alpha}{2}, n-1}}{\sqrt{n}}, & \sigma \text{ unknown} \end{cases}$

and set in $E(|S|)$.