Mandatory Exercise 2: tips and common mistakes

In general: Keep in mind that all the small questions in a numbered task count to the total score.

1.1.–1.3. Most of you did these tasks rather good.

1.1.) $X+Y$ – random variable

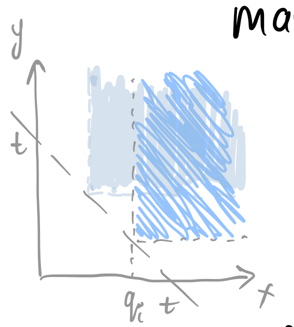
Only few explained this well and looked to understand

The point was that, because there is countably many rational numbers, for any real number t, the open halvplane

$$(X+Y \leq t)^c = (X+Y > t)$$

may be expressed as

$$\bigcup_{i=0}^{\infty} \underbrace{\big((X > q_i) \cap (Y > t - q_i)\big)}_{\text{event}}, \text{ where}$$



q_i is a rational number with number i.

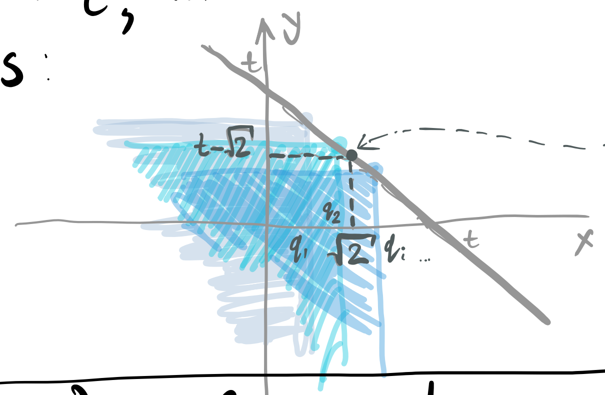
Hence $X+Y \leq t$ is also an event, as a complement to the event.

$\square$

! Note that we cannot express $X+Y \leq t$ directly as

$$\bigcup_{i=1}^{\infty} \left((X \leq q_i) \cap (Y \leq t - q_i) \right),$$

Because this union does not include uncountably many points on the line $X+Y=t$, in which X takes irrational values:



this point is not in this union

the set of random variables is a vector space

Please remember from a pure math course or read what is a vector space.

There were many possible solutions
You could explain that it is a subset of a vector space which is closed under addition and multiplication.
Or just refer to the fact that the set of functions $\Omega \to \mathbb{R}$ is a vector space. Or mention

that the set is close under addition and multiplication, includes identity elements, satisfies additivity, associativity, distributivity.

Many of you only showed that the set was closed under addition and multiplication. Or actually showed that the set was a group. Or something in-between. That was not enough.

1.2. Most of the answers were well. However, some of you did not answer to all the small questions and therefore did not get the full score.

2. The notation for parameter in gamma distribution varies: some denote $\frac{1}{\beta}$ what the others denote β. The task was to find the density of βY, where $Y \sim \text{Gamma}(\alpha, 1)$. Such way it is clearly defined what is β here.

To 'find' meant 'obtain an expression', 'derive a formula', and **NOT** 'copy from some source'

The density could be found by applying scale transformation (§2.1 or §3.5 in Cassella, Berger).

The moment generating functions could be found by further integration.

3.1. completeness

The theorem 6.2.25 was useful.

Quite many tried to show competeness by definition, by writing down something like

$$0 = E(g(t(x))) = \text{const}(x) \cdot \int_0^\infty g(t(x)) \cdot x^{\alpha-1} \cdot e^{-\frac{x}{\beta}} \, dx$$

and claimed that as const (x), $x^{\alpha-1}$ and $e^{-\frac{x}{\beta}}$ are positive, then $g(t(x)) = 0 \ \forall x > 0$ This claim should be explained, because the function g could take negative values for some x and positive for other X (like it happened f. ex. in problems 6.10 and 6.15 from exercise 3).

(Only one student grounded it further by uniqueness of Laplace transformation
This was a great attempt, and very near the proof of theorem 6.2.25 — such creative solutions count very positively! However, in that solution the function g included the unknown parameter, so that proof failed too)

illustrate the level sets

Remember that $X > 0$, i.e. the gamma distributed random variable takes only positive values

conditional distribution

It was ok to find the conditional distribution up to the normalizing constant (which could be done by writing the joint density and attentive looking to the formula).
In 3.1., you could find the whole expression for the conditional density together with the constant by tools of §4.2. in Cassela&Berger. In 3.3., 3.4 finding the normalizing constant was too challenging, some general explanation here was enough.

3.3, 3.4 Those who answered most of 3.1 but skipped 3.3., 3.4 got some score for the small questions where the answers were identical — but no score when the answeres differed a little