TMA 4115 IF: komplekse tall

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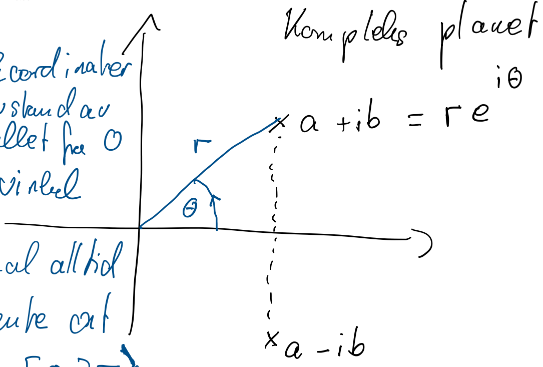


Polarkoordinater

r = avstand av tallet fra 0

θ = vinkel

Vi skal alltid forvente at $\theta \in [0, 2\pi\rangle$

Produkt symbol

Idé: Symbol som summetegn men for produkter

$$\sum_{i=1}^{2} z_i = z_1 + \sum_{i=2}^{2} z_i = z_1 + z_2$$

$$\prod_{i=1}^{2} z_i = z_1 \cdot \prod_{i=2}^{2} z_i = z_1 \cdot z_2$$

i algebras fundamentalteorem

$$\prod_{k=1}^{n} (z - z_k) = (z - z_1) \cdot (z - z_2) \cdot \ldots \cdot (z - z_n)$$

Algebras fundamentalteorem sier at vi kan alltid skrive en polynom som en produkt av enkle 1st. grads polynomer med komplekse tall

→ går ikke med reelle tall

$$z^2+1 \neq (z+a)(z+b)$$

for $a, b \in \mathbb{R}$

Hvordan lager man produktet?

⇝ Finn nullpunkter!!

$$z^2+1=0$$

$$z^2=-1 \quad \Rightarrow z=\pm i$$

$$z^2+1 = \underbrace{(z+i)(z-i)}_{z^2+\cancel{iz}-\cancel{iz}+\underbrace{i(-i)}_{=1}}$$

Deling av komplekse tall

$$\frac{a+ib}{c+id}=\frac{a+ib}{c+id}\cdot\frac{c-id}{c-id}$$

$$=(a+ib)(c-id)$$

$$\boxed{(x+y)(x-y) = x^2 - y^2} \longrightarrow \boxed{\overline{(c+id)}\,(c-id)}$$

$$= \frac{(a+ib)(c-id)}{\underbrace{c^2 - i^2d^2}_{c^2+d^2}}$$

$$= \frac{ac - i(ad - bc) + bd}{c^2+d^2}$$

$$= \frac{ac+bd}{c^2+d^2} - \frac{(ad-bc)}{c^2+d^2}\, i$$

Kvadratrøtter

For positive reelle tall a, b gjelder

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

gjelder ikke generelt:

$$1 = \sqrt{1} = \sqrt{(-1)\cdot(-1)}$$

$$\textcircled{=} \sqrt{-1} \cdot \sqrt{-1} = i \cdot i = -1$$

$$\lightning$$

Konjugering

$$z = a + ib$$

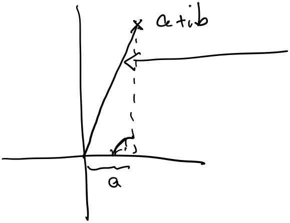
$$\bar{z} = a - ib$$

$$a + ib = z = \bar{z} = a - ib$$

$$\iff ib = -ib \iff b = 0$$

Hvis $z = 0 + ib$

$$\bar{z} = -ib = -z$$



lengden betegner vi med $|a+ib|$

$$|a+ib| = \sqrt{a^2+b^2}$$

$$a + ib = re^{i\varphi}$$

$$|re^{i\varphi}| = r$$

Oppgaver

$$i^{17} = i^{16+1} = \underbrace{i^{16}}_{=1} \cdot i = i$$

$$|i| = 1$$

$$\|$$

$$0 + 1 \cdot i$$

$$|0 + 1 \cdot i| = \sqrt{0^2 + 1^2}$$

$$\bar{i} = -i$$

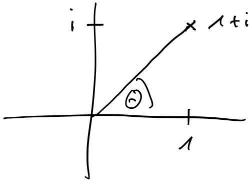
$$(1+i)^6 = \underbrace{(1+i)(1+i)}\underbrace{(1+i)(1+i)}_{2i}\underbrace{(1+i)(1+i)}_{2i}$$

$$\cancel{1} + \underbrace{i\cdot 1 + 1\cdot i}_{\boxed{2i}} + \underbrace{i^2}_{=\cancel{1}}$$

$$= (2i)^3 = 2^3 \cdot i^3 = \underline{\underline{-8i}}$$

$$|-8i| = 8 \qquad \overline{-8i} = 8i$$

$1+i$



$$|1+i| = \sqrt{2}$$

$$\theta = \frac{\pi}{4}$$

$$1+i = \sqrt{2}\, e^{i\frac{\pi}{4}}$$

$$(1+i)^6 = \left(\sqrt{2}\, e^{i\frac{\pi}{4}}\right)^6 = \sqrt{2}^6 \cdot \underbrace{\left(e^{i\frac{\pi}{4}}\right)^6}$$

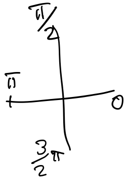
$$e^{i\theta} \cdot e^{i\varphi} = e^{i(\theta+\varphi)} \qquad e^{i\frac{\pi}{4}\cdot 6}$$

$$= \sqrt{2}^6 \cdot e^{i\frac{3}{2}\pi}$$

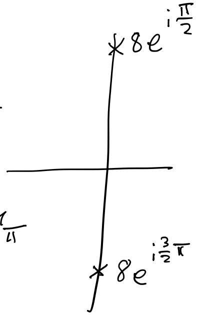
Hesh: $\sqrt{2} = 2^{1/2}$

$$(\sqrt{2})^6 = (2^{1/2})^6$$
$$= 2^3 = 8$$

$$= 8\underbrace{e^{i\frac{3}{2}\pi}}_{-i} = -8i$$



$$8e^{i\frac{3}{2}\pi}$$

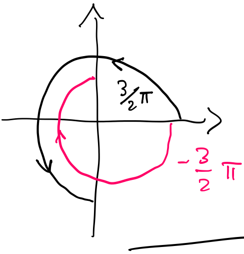


$$\overline{8e^{i\frac{3}{2}\pi}} = 8e^{i\frac{1}{2}\pi}$$

Enkelt måte å finne den konjugerte for et komplekse tall i polarform

Idé: $\frac{3}{2}\pi + \frac{1}{2}\pi = 2\pi$

$$\Leftrightarrow \underbrace{2\pi - \frac{3}{2}\pi}_{\text{Vi kan tolke vinkelen som en vinkel } -\frac{3}{2}\pi} = \frac{1}{2}\pi$$



$$\overline{8e^{\frac{3}{2}\pi i}} = 8e^{\boxed{-\frac{3}{2}\pi i}}$$

går ikke an i polarform fordi vi skal ha vinkler mellom 0 og 2π

Kan enkelt rettes ved å lage

$$-\frac{3}{2}\pi + 2\pi$$

$$i^5 + i + 1 = i + i + 1 = 1 + 2i$$

$$(3+3i)^8 = (3(1+i))^8 = 3^8(1+i)^8$$

$$= 3^8(1+i)^{6+2}$$

$$= 3^8\underbrace{(1+i)^6}_{-8i}\underbrace{(1+i)^2}_{2i}$$

$$= 16\cdot 3^8 + 0\cdot i$$