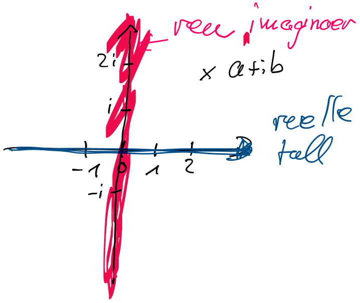
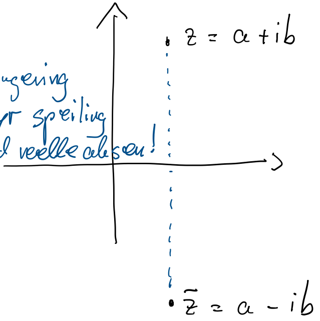
TMA 4115 IF

Friday, 12 January 2024 11:59

$$i = \sqrt{-1}$$
$$i^2 = -1$$
$$i^3 = i \cdot i^2 = -i$$
$$i^4 = 1$$
$$i^5 = i$$



Konjugering betyr speiling ved reelle aksen!

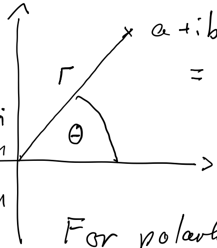


Polarkoordinater

r = avstand av punkt fra 0

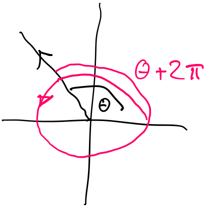
θ vinkel som vi pleier å gi som et vinkel mellom

$[0, 2\pi)$



$$a + ib = re^{i\theta}$$

For polarkoordinater spiller det ingen roll om vi bruker θ eller $\theta + 2\pi$



Algebra's fundamental teorem

Hver polynom kan skrives som et produkt av 1st. grads polynomer

Produkt symbolet: $\prod_{i=1}^{n}$

$\sum$ summetegn

$$\sum_{i=1}^{2} a_i = a_1 + \sum_{i=2}^{2} a_i = a_1 + a_2$$

$$\prod_{i=1}^{2} a_i = a_1 \cdot \prod_{i=2}^{2} a_i = a_1 \cdot a_2$$

$$\prod_{k=1}^{n} (z - z_k) = (z - z_1)(z - z_2) \cdots \cdot (z - z_n)$$

Merk: For reelle tall går det vanligvis ikke

$$\boxed{z^2 + 1} = (z+a)(z+b)$$

hvor a og b er nullpunkter til polynomet

$$0 = z^2+1 \iff z^2 = -1$$

$z = i$ eller $z = -i$

Fun. teorem:

$$z^2+1 = (z+i)(z-i)$$

$$z^2 \cancel{-iz} \cancel{+iz} - \underbrace{i\cdot i}_{-1}$$

lurt her z^2+1

$$\frac{a+ib}{c+id} = \frac{a+ib}{c+id}\cdot\boxed{\frac{c-id}{c-id}} = \frac{(a+ib)(c-id)}{\boxed{(c+id)(c-id)}}$$

$$\boxed{(x+y)(x-y) = x^2-y^2}$$

$$= \frac{ac - aid + ibc + \boxed{ib(-id)}}{c^2+d^2}$$

$+bd$

$$(c+id)(c-id)$$

$$= c^2 - \underbrace{(id)^2}_{i^2d^2} = c^2+d^2$$

$$= \frac{ac+bd}{c^2+d^2} + i\frac{bc-ad}{c^2+d^2}$$

For x, y positive reelle tall så gjelder

$$\sqrt{xy} = \sqrt{x}\sqrt{y}$$

$$1 = \sqrt{1} = \sqrt{(-1)(-1)} \overset{\lightning}{=} \sqrt{-1}\cdot\sqrt{-1}$$

$$= i\cdot i = -1$$

$$z = a+ib$$

$$\bar{z} = a-ib$$

$$\cancel{a}+ib = z = \bar{z} = \cancel{a}-ib$$

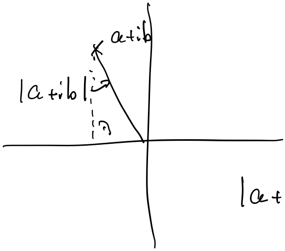
$$(2ib = 0 \quad \Rightarrow \quad b = 0$$

$\Rightarrow$ $z = a$ er ett reell tall

Hvis $w = bi$ (ren imaginær)

$$\bar{w} = -bi = -w$$

Absolutt verdi (avstand av tallet til 0)



$$|re^{i\theta}| = r$$

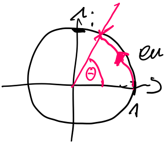
$$|a+ib| = \sqrt{a^2+b^2}$$

$$i^{17} = i^{16+1} = i \cdot \underbrace{i^{16}}_{=1} = i$$

$$\overline{0+1\cdot i} = 0-1\cdot i$$

$$|i| = 1 \quad \bar{i} = -i \quad \Big\lfloor i = 0+1\cdot i$$

$$|i| = \sqrt{0^2+1^2}$$



$$e^{i\theta}$$

$$|e^{i\theta}| = 1$$

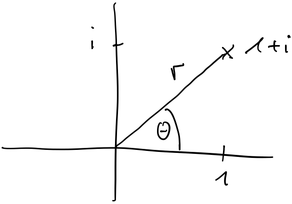
$$(1+i)^6 = (1+i)\cdot(1+i)\underbrace{(1+i)(1+i)}_{2i}\underbrace{(1+i)(1+i)}_{2i}$$

$$\underbrace{1+i+i+i^2}$$

$2i$

$= (2i)^3 = 2^3 \cdot i^3 = -i8 = -8i$

$\overline{-8i} = 8i \qquad |-8i| = \sqrt{0^2+(-8)^2} = 8$



$r = \sqrt{1^2+1^2}$

$= \sqrt{2}$

$\theta = \frac{\pi}{4}$

$1+i = \sqrt{2}e^{i\frac{\pi}{4}}$

$(1+i)^6 = \left(\sqrt{2}\, e^{i\frac{\pi}{4}}\right)^6 = \sqrt{2}^6 \cdot \left(e^{i\frac{\pi}{4}}\right)^6$

$e^{i\varphi} \cdot e^{i\theta} = e^{i(\varphi+\theta)}$

$= \sqrt{2}^6 \cdot e^{i\frac{\pi}{4}\cdot 6}$

$= 8 \cdot \underbrace{e^{i\frac{3\pi}{2}}}_{=-i}$

$\sqrt{2} = 2^{1/2}$

$(\sqrt{2})^6 = \left(2^{1/2}\right)^6 = 2^{6/2}$

$= 2^3$

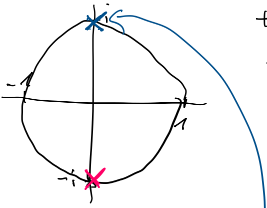
$= 8$

$e^{i\frac{3}{2}\pi} = \cos\frac{3}{2}\pi$

$+ i \sin\frac{3}{2}\pi$

$= -i$

$\boxed{e^{\frac{3}{2}\pi i}}$



$$e^{\frac{3}{2}\pi i} = e^{\frac{\pi}{2} i}$$

$$\frac{3}{2}\pi + \frac{1}{2}\pi = 2\pi$$

Euler formel for konjugerte til komplekse tall

$$r e^{i\theta} = r e^{i(2\pi - \theta)}$$

Husk: $-\theta$ er for polarformen det samme som $2\pi - \theta$

$$z = \underbrace{i^5}_{=i} + i + 1 = 2i + 1$$

$$8 = 6 + 2$$

$$(3+3i)^8 = (3(1+i))^8 = 3^8 (1+i)^8$$

$$= 3^8 \underbrace{(1+i)^6}_{-8i} \underbrace{(1+i)^2}_{2i}$$

$$= 3^8 \cdot 8 \cdot 2 + 0 \cdot i$$